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INDIAN ASSOCIATION OF PHYSICS TEACHERS
NATIONAL STANDARD EXAMINATION IN ASTRONOMY 2013-2014

Date of Examination: 24th November 2013

Time 15.00 to 17.00 Hrs

	4	8	5
Q. P. Code No.	FOUR	EIGHT	FIVE

Please read the instructions carefully before answering

INSTRUCTION TO CANDIDATES

1. On the answer sheet fill up all the entries carefully in the space provided, **ONLY IN BLOCK CAPITALS**. Use only **BLUE OR BLACK BALL PEN** for making entries and marking answers. Incomplete / incorrect / carelessly filled information may disqualify your candidature.

2. Write Q.P. Code No. mentioned above on your answer sheet (in the space provided). Otherwise your answer sheet will NOT be valued.

3. The question paper has 80 multiple-choice questions. Each questions has 4 options, out of which only one is correct. Choose the correct answer and shade the oval in the corresponding box on the answer sheet as shown below :

Correct method

● ○ ○ ○

4. A correct answer carries 3 marks and 1 mark will be deducted for each wrong answer.

5. All rough work may be done on the blank sheet provided at the end of question paper.

6. ***PLEASE DO NOT MAKE ANY MARKS OTHER THAN SHADING IN THE SPACE PROVIDED ON THE ANSWER SHEET.*** Answer sheets are evaluated with the help of a machine. Due to this, **CHANGE OF ENTRY IS NOT ALLOWED.**

7. **Scratching or overwriting may result in wrong score. DO NOT WRITE ANYTHING ON THE BACK OF ANSWER SHEET.**

8. Use of a nonprogrammable calculator is allowed.

9. **The answers/solution to this question paper will be available on our website - www.iapt.org.in by 30th November 2013.**

CERTIFICATES & AWARDS :

i) Certificate for top 10% students of each centre.

ii) Merit certificates to statewise Top 1% students.

iii) Merit certificate to Nationwise Top 1% students.

10. Result sheets and the "centre top 100%" certificates of NSEJS are dispatched to the professor in charge of the centre. You will get your marks from the professor in charge of your centre by January 2014 end.

11. 300 (or so) students are selected for the next examination - Indian National Junior Science Olympiads (INJSO). Individual letters are sent to these students only

12. No queries will be entertained in this regard.

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1. When an asteroid is assigned a number in front of its name, what additional information does that provide?

(a) The year of discovery
(b) Its orbit has been calculated
(c) Its distance from the sun
(d) Its number in the list of discovered asteroids

Ans. **(b)**

2. When Venus has reached its maximum eastern elongation from the sun as viewed from the earth, it is visible in the sky

(a) in opposition to the sun
(b) as an evening ''star''.
(c) as a morning ''star''
(d) in conjunction with the sun.

Ans. **(b)**

3. In a transistor, a change in emitter current by 8.0 mA produces a change of 7.8 mA in the collector current. What change in the base current produces the same change in the collector current.

(a) 50 μA (b) 100 μA (c) 150 μA (d) 200 μA

Sol. **(d)**

$\Delta E = \Delta I_C + \Delta I_B$

$= 8\text{ m A} = 7.8\text{ mA} + \Delta I_B$

$\Delta I_B = 0.2\text{ mA}$

$= 200\ \mu A$

4. An object is placed at a distance of 16 cm from a lens and an image is produced. If the object is moved towards the lens by 10 cm another image of same magnification is produced. Then lens must be :

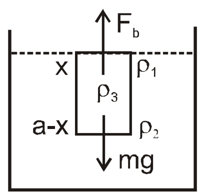
(a) Convex
(b) Concave
(c) Two concave lens in contact
(d) Convex lens of higher focal length in contact with concave lens of short focal length.

Sol. **(a)** Real object and Real image
Real object and Virtual image

5. A solid homogeneous cube floats at the interface of mercury and water, such that the lower portion of the cube is immersed in mercury and the upper portion is in water. The density of the material of the cube is 7.3 gm/cc and those of mercury and water are respectively 13.6 gm/cc and 1.0 gm/cc. Then the % volume of the cube immersed in mercury is :

(a) 70% (b) 68% (c) 50% (d) 73%

Sol. **(c)** $F_b = mg$



$\rho_1 a^2 x g + \rho_2 a^2 (a - x)g = \rho_3 a^3 g$

$\rho_1 x + \rho_2 (a - x) = \rho_3 a$

$1x + 13.6 a - 13.6 x = 7.3 a$

$13.6 a - 7.3 a = 13.6 x - x$

$6.3 a = 12.6 x$

$$x = \left(\frac{6.3a}{12.6}\right)$$

$$\%V = \left(\frac{a-x}{a}\right) = \frac{a - \frac{6.3a}{12.6}}{a}$$

$$= \frac{126 - 6.3}{12.6} = \left(\frac{6.3}{12.6}\right) \times 100$$

$= 50\%$ .....(C)

6. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then

(a) p = 2, q = – 7 (b) p = – 2, q = 7 (c) p = 2, q = – 7 (d) p = 2, q = 7

Sol. **(a)** $y = 4x - 5$

$y^2 = px^3 + q$

$9 = 8p + q$ satisfy (2,3) ....(1)

and $\frac{dy}{dx} = 4$

and $2y\frac{dy}{dx} = 3px^2 + 0$

$2(3)(4) = 3p(2)^2$
$p = 2$
$q = 9 - 8(2) = 9 + 16$
$q = -7$

7. It is given $S = \frac{a}{b}+\frac{c}{d}+\frac{1}{e}$. If $0 < a < b < c < d < e$ in the equation, then the greatest increase in S would result from adding 1 to the value of which variable ?
(a) a (b) b (c) c (d) d

Sol. **(a)** Question is incomplete here $1 < a < b < c < d < e$ should be then answer is (a)

$S' = \frac{a+1}{b}+\frac{c}{d}+\frac{1}{e}$

$S' = \frac{1}{b} + S$ $\qquad$ $b < d \Rightarrow \frac{1}{b} > \frac{1}{d}$

$S'' = \frac{1}{d} + S$

8. The largest moon in our solar system has an atmosphere that is denser that the atmosphere of Mars. The name of this moon is :
(a) Titan (b) Ganymede (c) Trition (d) Io

Ans. **(a)**

9. A metal beam of length 30m is standing vertically on a floor in an unstable equilibrium. If the floor prevents the beam forms slipping then the top end of the beam hits the ground with a speed in the units of km per hour is (take $g = 10\ m/s^2$)
(a) 108 (b) 54 (c) 36 (d) 81

Sol. **(a)** $mg\frac{\ell}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$

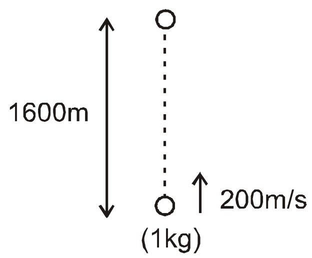
$\omega = \sqrt{\frac{3g}{\ell}}$

$V = \omega\ell = \sqrt{3g\ell} = \sqrt{3x10x30} = 30\ m/s$

$v = 30 \times \frac{18}{5} = 108\ km/hr$

10. A body of 1 kg is projected vertically upward with a speed of $200\ ms^{-1}$. It rises to a height of 1600 m. If gravitational field is uniform and equal to $10\ Nkg^{-1}$, then the energy used up in overcoming the air resistance and the average thrust on the body due to this resistance respectively are :
(a) 4kJ, 2.5 N (b) 5kJ, 10 N (c) 5kJ, 2.5 N (d) 4kJ, 10 N

Sol. **(a)**



$W_g + W_{air} = \Delta k \Rightarrow \quad -1 \times 10 \times 1600 + w_{air} = -\frac{1}{2} \times 1 \times (200)^2$

$\Rightarrow \quad -1600 + w_{air} = -20000$

$\Rightarrow \quad w_{air} = 4000$ J

$\therefore \quad = -4$kJ

So energy used in overcoming air resistance = 4 kJ

Now F × 1600 = 4000 $\Rightarrow \quad F = \frac{4000}{1600} = 2.5$N

11. Annular eclipse may be observed :
(a) when the sun is in the perihelion
(b) when the sun is at the appealion
(c) during the winter solicits
(d) during the summer solicits

Ans. **(a)**

12. A parallel-plate capacitor is charged and then disconnected from the battery. If the plates are then moved farther apart by the use of insulated handles, then
(a) The charge on the capacitor decreases
(b) The capacitance on the capacitor increases
(c) The voltage across the capacitor does not change
(d) The voltage across the capacitor increases

Sol. **(d)** $C = \frac{A\varepsilon_0}{d}$ As d↑ C↓

$V = \frac{Q}{C}$ as C↓, V↑

13. A coil of diameter 2.0 cm has 300 turns. If the coil carrying a current of 10 milliamps is immersed in a constant magnetic field of 0.05 Tesla, the maximum torque that could b e exerted on the coil by the magnetic field is :
(a) 4.7 Nm (b) 4.7×10^{-5} Nm (c) 4.7×10^{-4} Nm (d) 4.7×10^{-8} Nm

Sol. **(b)** $\tau_{max} = \pi r^2$ NiB

$= \pi \times 10^{-4} \times 300 \times 10 \times 10^{-3} \times \frac{0.05}{100}$

$= \pi \times 10^{-6} \times 3 \times 5$

$= 4.7 \times 10^{-5}$ Nm

14. When an ideal monatomic gas is heated at constant pressure, the ratio of the increase in internal energy to the heat energy supplied is :
(a) 2/5 (b) 3/5 (c) 2/7 (d) 3/7

Sol. **(b)** $\Delta U = nC_V\Delta T$. $\Delta Q = nC_P\Delta T$.

$\frac{\Delta U}{\Delta Q} = \frac{C_V}{C_P} = \frac{3}{5}$

15. In the expansion of $\left(\sqrt{2} + \sqrt[3]{5}\right)^{20}$ the number of rational terms will be
(a) 6 (b) 10 (c) 4 (d) 16

Sol. **(c)**

$\left(\sqrt{2} + \sqrt[3]{5}\right)^{20} = (2^{1/2} + 5^{1/3})^{20}$

$= {}^{20}C_r.(2^{1/2})^{(20-r)}.(5^{1/3})^r$

$= {}^{20}C_r\, 2^{20-r/2}.5^{r/3}$

$\frac{20-r}{2}$ is integer r = 0, 2, 4, ... 20

$\frac{r}{3}$ is integer r = 0, 3, 6, 9, 12, 15, 18.

common values of r = 0, 6, 12, 18 four terms

16. Consider the equation $\frac{11x^2+33x+15}{22x^2+33x-8}=\frac{x+3}{2x+3}$. The number of roots of this equation is

(a) 4 (b) 3 (c) 2 (d) 1

Sol. **(d)**

$\frac{11x^2+33x+15}{22x^2+33x-8}=\frac{x+3}{2x+3}$

$\Rightarrow$ 23x = – 69 $\Rightarrow$ x = – 3 only solution

17. If cos 10º – sin 10º = k, then cos 20º is

(a) $\sqrt{2k-k^2}$ (b) $k\sqrt{2-k^2}$ (c) $\frac{k}{\sqrt{2-k^2}}$ (d) $k+2\sqrt{2-k^2}$

Sol. **(b)**

cos 10º – sin 10º = k

Squaring

$\cos^2 10^\circ + \sin^2 10^\circ - 2\sin 10^\circ \cos 10^\circ = k^2$

$\Rightarrow$ $1 - \sin 20^\circ = k^2$

$\Rightarrow$ $\sin 20^\circ = 1 - k^2$

$\cos 20^\circ = \sqrt{1-(\sin 20^\circ)^2}$

$= \sqrt{1-(1-k^2)^2}$

$= \sqrt{1-(1+k^4-2k^2)}$

$= \sqrt{2k^2-k^4}$

$= k\sqrt{2-k^2}$

18. If $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$ then, x is equal to

(a) (a + b + c) (b) –(a + b + c) (c) (ab + bc + ca) (d) $-(a^2+b^2+c^2)$

Sol. **(b)**

$C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$

$\Rightarrow$ (x+ a + b + c)

$\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$

then x + a + b + c = 0 $\Rightarrow$ x = – (a + b + c)

19. If a,b,c are in A.P., x is GM of a and b while y is GM of b and c, then b^2

(a) AM of x^2 and y^2 (b) GM of x^2 and y^2 (c) HM of x^2 and y^2 (d) None of these

Sol. **(a)**

a, b, c are in A.P.

$2b = a + c$ .....(1)

x is G.M.of a and b $\Rightarrow$ $x = \sqrt{ab}$ $\Rightarrow x^2 = ab$

y is G.M.of b and c $\Rightarrow$ $y = \sqrt{bc}$ $\Rightarrow y^2 = bc$

Now $x^2 + y^2 = (a + c)b$

$x^2 + y^2 = (a + c)b$ $\Rightarrow$ $\frac{x^2 + y^2}{2} = b^2$

A.M. of x^2 and y^2 is b^2

20. What is the smallest angular separation v of a double star which can be so recognized by a telescope with an aperture of 50 inches (diameter). Assume an effective wavelength of 540 nm.
(a) v = 4.3×10^{-8} radians (b) v = 5.2×10^{-7} radians
(c) v = 1.8×10^{-5} radians (d) v = 5.4×10^{-5} radians

Sol. **(b)** $\theta = \frac{1.22\lambda}{D}$

21. Amalthea is the name of :
(a) An asteroid (b) The largest of the smaller moon of the Jupiter
(c) The largest moon of the Uranus d) A moon of the Saturn

Ans. **(b)**

22. The bary centre of the earth and the moon is located on a line joining them and is :
(a) About 1700 km away from the moon
(b) About 1700 km inside from the moon
(c) About 1700 km away from the surface of the earth
(d) About 100000 km away from the earth

Ans. **(b)**

23. The Cassini division refers to :
(a) A particular type of mathematical operation used by the astronomers
(b) The distance between the Mars and Jupiter
(c) The gap between the first and the second ring of the Saturn
(d) The distance between the surface and the ring of the Uranus

Ans. **(c)**

24. In the asteroid belt, some areas are almost free of asteroids. These are known as,
(a) Kirkwood's gaps (b) Roche's gaps (c) Cassini division (d) Kuiper's gaps

Ans. **(a)**

25. All dwarf planets,
(a) Belong to the Kuiper belt
(b) Are smaller than the Earth's moon
(c) Were discovered during the last two decades
(d) Are either equal to or less that the size of Pluto

Ans. **(a)**

26. As viewed from the earth most stars appear to move across the sky because
(a) the earth revolves round the sun
(b) the earth spins about its own axis
(c) stars are very distant objects
(d) stars revolve around the centre of the galaxy

Ans. **(b)**

27. Galileo discovered something about Venus with his telescope that shook the old theories. Which of the following was Galileo's discovery ?
(a) Venus was covered in clouds (b) Venus has phases like the moon
(c) Venus' surface was similar to the earth's (d) Venus had retrograde motion

Ans. **(b)**

28. Name the phase of the moon in case of lunar and solar eclipse
(a) Full moon for both phases (b) New moon for both phases
(c) Full moon for lunar and new moon for solar (d) New moon for lunar and full moon for solar

Ans. (c)

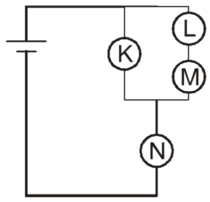
29. The phenomenon that causes the Moon's rotation about it's own axis to be equal to its period of revolution about the Earth is called
(a) Haner Effect (b) Landow Effect (c) Orbital Synchronization (d) Tidal locking

Ans. (d)

30. The comet known as Halley's Comet has an average period of :
(a) 56 years (b) 66 years (c) 76 years (d) 86 years

Ans. (c)

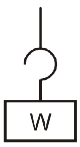
31. Four identical bulbs K, L, M, N are connected as shown in the circuit below are glowing. The bulb K is now taken out. Which of the following statements is correct ?

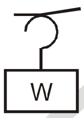


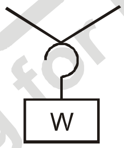
(a) all the bulbs will now blow off (b) bulb N glows more brightly
(c) bulb N becomes dimmer (d) bulb N glows same as it were earlier

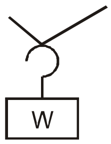
Ans. (c)

32. Equal weight has been suspended in different ways by strings as shown in the figures. In which case the possibility of breaking the string is the maximum.

(a) 

(b) 

(c) 

(d) 

Ans. (a)

33. An FM radio station transmits at a frequency of 102.8 MHz. Given Planck Constant h = 6.626 × 10^{34} J-s, the energy in eV of each of the photons emitted by the transmitter is :
(a) 6.6446×10^{-4} (b) 6.812×10^{-6} (c) 4.252×10^{-7} (d) 3.014×10^{-8}

Sol. (c) $E = h\nu = \dfrac{6.626\text{x}10^{-34}\text{x}102.8\text{x}10^{6}}{1.6\text{x}10^{-19}}$

$= 4.252 \text{ x } 10^{-7}$

34. At given instant of time, two radioactive elements P and Q have the same number of atoms. It is also known that the mean-life time of P is the same as the half-life time of Q, then which of the following is correct ?
(a) The decay rate of P is larger than that of Q
(b) The decay rate of Q is larger than that of P
(c) The decay constant of P is less that Q.
(d) The decay rate of P and Q will be same initially

Ans. (a)

35. If E = energy, G = gravitational constant , I = impulse and M = mass, the dimension of $\dfrac{GIM^2}{E^2}$ is same as that of :
(a) mass (b) length (c) time (d) force

Sol. (c) $\left[\dfrac{GIM^2}{E^2}\right] = \dfrac{\left[M^{-1}L^3T^{-2}\right]\left[MLT^{-1}\right]\left[M^2\right]}{\left[M^2L^4T^{-4}\right]}$

$= [T]$

36. A spherical black body of radius 12cm radiates 4.5 W of power when its surface temperature is 500 K. It the radius is halved and its temperature doubled, the power radiated in watt is

(a) 2.25 (b) 4.5 (c) 9.0 (d) 18.0

Sol. **(d)** $4.5 = \sigma \pi r^2 T^4$

$$E = \sigma \pi \left(\frac{r}{2}\right)^2 (2T)^4$$

E = 18 watt

37. A train approaching a railway platform with a speed of 20 m/s starts blowing the whistle with a frequency of 640 Hz. If speed of sound in air is 340 m/s then frequency of the whistle as measured by a person standing on the platform is

(a) 600 Hz (b) 640 Hz (c) 680 Hz (d) 720 Hz

Sol. **(c)** $f' = 640 \left(\frac{340}{340 - 20}\right) = 680$ Hz

38. When a certain metal surface is illuminated with light of frequency f the stopping potential for the photoelectric current is V_0. When the same surface is illuminated with light of frequency f/2, the stopping potential is $V_0/4$. The threshold frequency for the photoelectric emission is

(a) $\frac{f}{6}$ (b) $\frac{f}{3}$ (c) $\frac{2f}{3}$ (d) $\frac{4f}{3}$

Sol. **(b)** $hf = w + eV_0$

$$h\frac{f}{2} = w + e\ \frac{v_0}{4}$$

$$hf - ev_0 = h\frac{f}{2} + e\frac{v_0}{4}$$

$$h\frac{f}{2} = \frac{3}{4}\ ev_0$$

$$hf = \frac{3}{4}\ eV_0$$

$$hf = w + ev_0$$

$$w = \frac{1}{3}hf$$

$$f_0 = \frac{f}{3}$$

39. In a thermal power plant electricity is generated by burning coal. The engine that works with maximum efficiency has to operate between the pressurized steam temperature 327ºC and environment temperature 27ºC. To produce one joule of electricity, the amount of heat energy lost to the environment in joule is.

(a) 0.0 J (b) 1.0 J (c) 0.5 J (d) 0.75 J

Sol. **(b)** Efficiency $= 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5$

Heat lost = 1 J

40. An object is kept at a distance 15 cm from a thin convex lens and a real image with magnification 2 is obtained. An identical lens is now placed in contact with the first one keeping the position of the object unchanged. The magnification of the new image thus formed is

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol. **(b)** $\frac{1}{30}-\frac{1}{(-15)}=\frac{1}{f}$

f = 10 cm.

Effective focal length when another identical lens placed

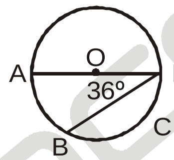
f_{eq} = 5 cm.

$\frac{1}{v}-\frac{1}{(-15)}=\frac{1}{5}$

v = $\frac{15}{2}$

m = $\frac{v}{u}=\frac{1}{2}$

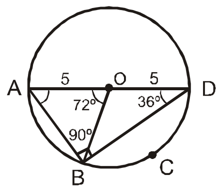
41. In the figure shown below, AD is a diameter of the circle with centre O and AO = 5 units. What is the length of arc BCD ?



(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 3π

Sol. **(d)**



$\theta=\frac{\ell}{r} \Rightarrow \ell=\theta r$

ℓ = $\overline{BCD}=\left(\frac{3\pi}{5}\right)\times 5 = 3\pi$

$\angle$BOD = 180º – 72º = 108º

$=\frac{3\pi}{5}$

42. If m and p are positive integers and (m + p) x m is even, which of the following must be true ?

(a) If m is odd, then p must be odd (b) If m is odd, then p must be even

(c) If m is even, then p must be even (d) If m is even, then p must be odd

Sol. **(a)**

(m + p) × m is even

If m is odd then m + p is even

$\Rightarrow$ p must be odd

43. In a collage of 300 students every student read 5 newspapers and every newspaper is read by 60 students. The number of newspapers in the college is

(a) at least 30 (b) at most 20 (c) exactly 18 (d) exactly 25

Sol. **(d)**

300 × 5 = 60 × x

x = $\frac{300\times 5}{60}$ = 5 × 5

x = 25

44. If P,Q,R are the angles of the triangle PQR, then the value of the determinant of the matrix is equal to

$$\begin{pmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{pmatrix}$$

(a) – 1 (b) 0 (c) 0.5 (d) 1

Sol. **(b)**

$= -1(1 - \cos^2 P) - \cos R\,(-\cos R - \cos P \cos Q) + \cos Q\,(\cos P \cos R + \cos Q)$

$= -1 + \cos^2 P + \cos^2 R + 2 \cos P \cos Q \cos R + \cos^2 Q$

$= -1 + \frac{3}{2} + \frac{1}{2}\,[\cos 2P + \cos 2Q + \cos 2R] + 2 \cos P \cos Q \cos R$

$= -1 + \frac{3}{2} + \frac{1}{2}\,[-1 - 4 \cos P + \cos Q + \cos R] + 2 \cos P \cos Q \cos R$

$= -1 + \frac{3}{2} - \frac{1}{2}$

$= 0$

45. A load of 1 kg is suspended by a thin wire having breaking strength of 25N. The load is allowed to fall when the wire is horizontal ($g=10\ ms^{-2}$). Then

(a) wire may break before it becomes vertical.

(b) the tension in the wire when the load is at lowest point is 20N

(c) system behaves like a pendulum

(d) the load loops a loop

Sol. **(a)** velocity at lowest position $v = \sqrt{2gL}$

$\frac{mv^2}{L} = T - mg$

$T = 30$ N.

46. Spherical bob of radius r of a simple pendulum is subjected to frictional force mostly due to air viscosity (with viscosity coefficient n) only and the viscous force is determined by stoke's law. Assuming the system to be lightly damped, the time t in which the amplitude of the pendulum becomes half of its original value is proportional to

(a) n (b) na (c) (a/n) (d) (n^2/a)

Sol. **(c)** $\frac{A_0}{2} = A_0 e^{\frac{-6\pi\eta r t}{2m}}$

$t \,\alpha\, \frac{1}{\eta} \quad \Rightarrow \quad t = \frac{a}{\eta}$

47. The escape velocity of a particle from the surface of earth, is 11km per second. If it is projected from a point at a altitude of 3R (R is the radius of the earth), then its escape velocity, in km per second will be

(a) 11 (b) 2.75 (c) 5.5 (d) 22

Sol. **(c)** $\frac{1}{2}\, m\,(11)^2 - \frac{GMm}{R} = 0$

$\frac{1}{2} m\,(11)^2 = \frac{GMm}{R}$ ...(1)

$\frac{1}{2}\, mv^2 = \frac{GMm}{4R}$ ...(2)

Divide (1) & (2)

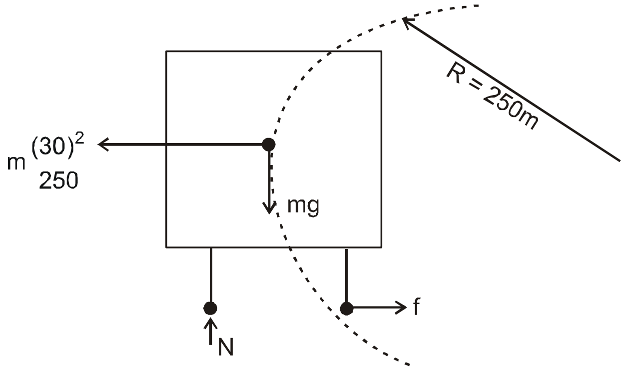
$\left(\frac{11}{v}\right)^2 = 4$

$v = 5.5$ km/s

48. A smooth horizontal curved road has an average radius of curvature of 250m. The centre of gravity of a car moving on the road is one meter above the ground. The car can move safely at a speed of 108 km per hour if its outer wheels are separated by

(a) 1m (b) 0.84 m (c) 0.72 m (d) 0.36 m

Sol. **(c)**



for no toppling $mg \times \frac{d}{2} = m \times \frac{(30)^2}{250}$

$d = \frac{900 \times 2}{250 \times 10} = \frac{18}{25} = 0.72$ m

49. A particle is moving with a constant velocity along a straight line parallel to the Y axis away from the origin. The magnitude of angular momentum with respect to the axis passing through and origin perpendicular to the palne of motion

(a) cannot be defined (b) remains constant
(c) goes on increasing with time (d) goes on decreasing with time

Ans. **(b)**

50. Two packs of playing cards (each containing 52 cards) are thoroughly mixed and two cards are taken at random one after the other without replacement. What is the probability that both of them are queens?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{7}{1339}$ (d) $\frac{1}{169}$

Sol. **(c)**

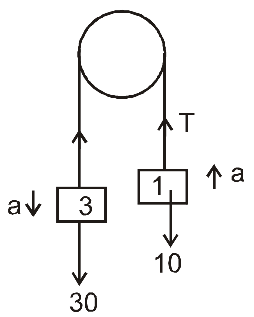
$P = \frac{8}{104} \times \frac{7}{103}$

$P = \frac{7}{1339}$

51. In an Atwood machine pulley and string are mass less and frictionless. If g = 10 ms^{-2}, and the two blocks involved have masses 3kg and 1 kg then the magnitude of the vertical acceleration of each of the blocks and that of the centre of mass of the system are

(a) $5ms^{-2}$, $5 ms^{-2}$ (b) $5ms^{-2}$, $2.5 ms^{-2}$ (c) $2.5ms^{-2}$, $2.5 ms^{-2}$ (d) $10ms^{-2}$, $10ms^{-2}$

Sol. **(b)**



30 – T = 3a

T – 10 = a

$$a_{CM} = \frac{5\times3-1\times5}{3+1} = 2.5\ \text{m/s}^2$$

52. Charon is orbiting round the

(a) Saturn (b) Jupiter (c) Pluto (d) Neptune

Ans. **(b)**

53. The bailey's beads are

(a) Some minor planets (b) Mountains on the Mars

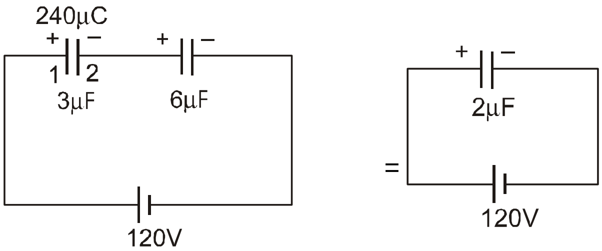
(c) A formation in a constellation (d) Phenomenon observed during the total solar eclipse

Ans. **(d)**

54. Two capacitors, one of 3 microfarads and the other of 6 microfarads, are connected in series and charged to a voltage of 120 volts. The potential difference, across the 3 microfarad capacitor is

(a) 80 V (b) 60 V (c) 40 V (d) 100 V

Sol. **(a)**



Q = 240 μc

So potential diff across 3μF capacitor = $\frac{240}{3} = 80$V

55. In the near infrared spectrum of carbon monoxide there is an intense band with a wave number of 2144 cm^{-1}. The fundamental vibration frequency of CO in Hz is about

(a) 6.43 × 1010 (b) 3.21 × 1012 (c) 6.43 × 1013 (d) 5.21 × 1014

Ans. **(c)**

56. A galvanometer having an armature coil with a resistance of 10 ohms requires 0.01 amperes for a full scale deflection. A resistance required to be connected in series to convert this galvanometer to a voltmeter of a range of 120 volts is

(a) 120 Ω (b) 1245 Ω (c) 1520 Ω (d) 11990 Ω

Sol. **(d)** $R_g = 10\ \Omega$, $I_g = 0.01$ A

Range of Voltmeter = 120 V

$V = I_g (R_g + R)$

120 = 0.01 (10 + R)

R = 12000 – 10 = 11990 Ω

57. When a particle oscillates in simple harmonic motion, both its potential and kinetic energy vary sinusoidally with time. If V be the frequency of the particle's motion the frequency associated with the kinetic energy

(a) 4v (b) 2v (c) v (d) v/2

Sol. **(b)** If v be the frequency of particle motion the frequency associated with the kinetic energy is 2v

58. If a photoelectric material has a work function of 4.00 eV, the maximum velocity of the photoelectrons emitted by light of energy 12.42 eV is (Take the mass of electron as 9.11×10^{-31} kg, 1eV = 1.60×10^{-19} joules)

(a) $1.72 \times 10^6\ ms^{-1}$ (b) $1.92 \times 10^8\ ms^{-1}$ (c) $1.98 \times 10^3\ ms^{-1}$ (d) $1.61 \times 10^4\ ms^{-1}$

Sol. **(a)** $K_{man} = hv - hv_0 = 12.42 - 4.00$

$$\frac{1}{2} mV^2_{man} = 8.42 \text{ ev}$$

$$V^2_{man} = \frac{2x8.42x1.6x10^{-19}}{9.1x10^{-31}} = 2.96 \times 10^{12}$$

$V_{man} = 1.72 \times 10^6$ m/s

59. Consider the motion of a planet of mass m in a circular orbit of radius r about the sun of mass M. Assuming the sun to be at rest, the total energy of the system is

(a) $E = \frac{GMm}{2r}$ (b) $E = -\frac{GMm}{2r}$ (c) $E = -\frac{GMm}{r}$ (d) $E = \frac{GMm}{r}$

Sol. **(b)** $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\Rightarrow$ $mv^2 = \frac{GMm}{r}$ ...(1)

$\text{T.M.E} = -\frac{GMm}{r} + \frac{1}{2}mv^2$

$= -\frac{GMm}{2r}$

60. An object of mass m_1 collides head on with a mass m_2, which is initially at rest. If the collision is perfectly inelastic (the coefficient of restitution = 0) then the fractional loss of kinetic energy of the whole system is

(a) $\frac{m_1}{m_1+m_2}$ (b) $\frac{m_2}{m_1+m_2}$ (c) $\frac{m_1 m_2}{(m_1+m_2)^2}$ (d) $\frac{4m_1 m_2}{(m_1+m_2)^2}$

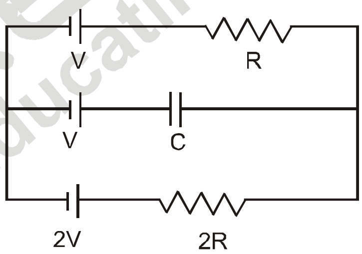
Sol. **(b)** $\frac{\Delta K}{K_i} = \frac{\frac{1}{2}(m_1+m_2)\left(\frac{m_1\mu}{m_1+m_2}\right)^2 - \frac{1}{2}m_1\mu^2}{\frac{1}{2}m_1\mu^2}$

$= \frac{m_1}{m_1+m_2} - 1$

$= \frac{m_2}{m_1+m_2}$

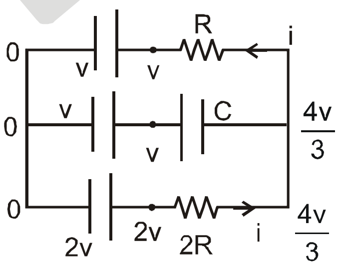
from certripetal fence

61. In the following circuit, the potential drop across the capacitor C must be



(a) V (b) $\frac{V}{2}$ (c) $\frac{V}{3}$ (d) $\frac{2V}{3}$

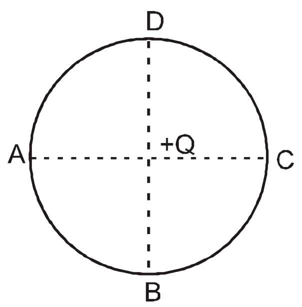
Sol. **(c)**



$i = \frac{v}{3R}$

Potential drop on capacitor $= \frac{4v}{3} - v = \frac{v}{3}$

62. Two particles of masses m and 4m have equal kinetic enrgies. Their de Broglie wavelengths are in the ratio of



(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

Sol. **(c)** $\lambda = \frac{h}{\sqrt{2mk}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$ frequal k.E.

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{4m}{m}} = 2$

63. A positive charge Q is located at the centre of a circle as shown in the following figure. W_1 is the work done in taking a charge of +3 C from A to B. W_2 Is is the work done in taking the same charge of +3 C from A to C. Then

(a) $W_1 < W_2$ (b) $W_1 = W_2/2$ (c) $W_1 > W_2$ (d) $W_1 = W_2 = 0$

Sol. **(d)** $V_A = V_B = V_c \Rightarrow W_1 = W_2 = 0$

64. What is the minimum number of NAND gates necessary for producing an AND gate?

(a) 1 (b) 2 (c) 3 (d) 4

Ans. **(b)**

65. In a hydrogen atom an electron of mass m and charge e is making n revolutions per second in an orbit of radius r. If the mass of the hydrogen nucleus be M, the magnetic moment associated with the orbital a motion of the electron is

(a) $\frac{\pi ner^2 m}{M}$ (b) $\frac{\pi ner^2 M}{m}$ (c) $\frac{\pi ner^2 m}{(M+m)}$ (d) πner^2

Sol. **(d)** w = $2\pi n$ rad/s $\Rightarrow T = \frac{1}{n}$

$c = \frac{e}{T} \Rightarrow M = c\pi r^2 = \pi ner^2$

66. The maximum value of $(15 \sin\theta + 8\cos\theta + 3)$ is

(a) 11 (b) 20 (c) 18 (d) 26

Sol. **(b)**

$15 \sin\theta + 8 \cos\theta + 3$

maximum value

$= \left(\sqrt{15^2 + 8^2}\right) + 3$

$= \sqrt{225 + 64} + 3$

$= \sqrt{289} + 3$

$= 17 + 3$

$= 20$

67. Two nonzero vectors **a** and **b** are such that **(a + b)** is perpendicular to **(a – b)**. Then
(a) a must be perpendicular to b and |a| must be equal to |b|.
(b) a must be perpendicular to b but |a| may not be equal to |b|.
(c) a must be perpendicular to b but |a| may be equal to |b|.
(d) a may be perpendicular to b and |a| must be equal to |b|.

Sol. **(d)**

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

$\Rightarrow \quad |\vec{a}|^2 - |\vec{b}|^2 = 0$

$\Rightarrow \quad |\vec{a}| = |\vec{b}|$ so and (d)

68. If a, b $\neq$ 1, ab > 0, a $\neq$ b and $\log_b a = \log_a b$, then ab = ?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 10

Sol. **(b)**

$\log_b a = \log_a b \Rightarrow \frac{\log a}{\log b} = \frac{\log b}{\log a}$

$\Rightarrow \quad (\log a)^2 = (\log b)^2$
$\Rightarrow \quad (\log a + \log b)(\log a - \log b) = 0$
$\log(ab) = 0 \quad \Rightarrow \quad ab = 1 \quad$ and $\quad \log a = \log b$
$\Rightarrow \quad a = b$
but a $\neq$ b

69. If a^2, b^2, c^2 are in arithmetic progression, then the terms $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ will form

(a) A.P. (b) G.P. (c) H.P. (d) none of these

Sol. **(d)**
a^2, b^2, c^2 are in A.P.
$2b^2 = a^2 + c^2$
$\Rightarrow \quad b^2 - a^2 = c^2 - b^2$

Now $\frac{2}{b+c} = \frac{1}{a+b} + \frac{1}{c+a}$

$$\frac{2}{b+c} = \frac{c+a+a+b}{(a+b)(c+a)}$$

$\Rightarrow \quad 2(ac + bc + a^2 + ab) = bc + 2ab + b^2 + c^2 + 2ac + bc$
$\Rightarrow \quad 2a^2 = b^2 + c^2$
$\Rightarrow \quad b^2, a^2, c^2$ are in A.P.
so order is change answer is (d)

70. Three point A,B and C have coordinates (a, b + c), (b, c + a) and (c, a + b)

(a) $a^2 + b^2 + c^2$ (b) $\frac{a^2+b^2+c^2}{2}$ (c) $\frac{a^2+b^2+c^2}{4}$ (d) zero

Ans. Question is incomplete according to option area of triangle A,B,C is asked so area of triangle ABC is 0. So answer is (d)

Sol. $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$C_1 \to C_1 \to C_2$

$= \begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix}$

$= (a + b + c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}$

$= C_1$ and C_3 are identical
$= 0$

71. The circles $x^2 + y^2 = 400$ and $x^2 + y^2 - 10x - 24y + 120 = 0$

(a) Do not intersect
(b) Intersect at two points
(c) Touch each other externally
(d) Touch each other internally

Sol. (d)

$x^2 + y^2 = 400$

$C_1(0, 0)$ $r_1 = 20$

$x^2 + y^2 - 10x - 24y + 120 = 0$

$(x - 5)^2 + (y + 12)^2 = 7^2$

C_2 (5, 12), $r_2 = 7$

$C_1C_2 = \sqrt{5^2 + 12^2} = 13$

$|r_1 - r_2| = |20-7| = 13$

Touch each other internally

72. In ΔABC, cosA + cosB + 2cosC = 2, therefore, the sides of ΔABC are in

(a) A.P.
(b) G.P.
(c) H.P.
(d) none of the above

Sol. (a)

$\cos A + \cos B + 2\cos C = 2$

$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2\,(1 - \cos C)$

$2\sin\frac{C}{2}\cos\left(\frac{A+B}{2}\right) = 2\left(2\sin^2\frac{C}{2}\right)$

$\dfrac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}} = 2$

$\Rightarrow \quad \dfrac{\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \dfrac{2}{1}$

Componendo and divendendo

$\dfrac{2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)}{2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)} = \dfrac{2+1}{2-1} = 3$

$\tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{3}$

$\sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}\sqrt{\dfrac{(s-a)(s-c)}{s(s-b)}} = \dfrac{1}{3}$

$\dfrac{s-c}{s} = \dfrac{1}{3} \quad \Rightarrow \quad 3s - 3c = s \quad \Rightarrow \quad 3c = 2s$

$3c = a + b + c$

sides are in A.P.

73. If $\dfrac{a^n - b^n}{a^{n-1} + b^{n-1}}$ is harmonic mean between a and b, the value of n is

(a) 0 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) 1

Ans. Data is wrong correct data is $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$. So answer is (a)

Sol. $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}} = \dfrac{2ab}{a+b}$

$\Rightarrow$ $a^{n+1} + ab^n + ba^n + b^{n+1} = 2b.a^n + 2ab^n$

$\Rightarrow$ $a^{n+1} - ab^n + b^{n+1} - ba^n = 0$

$\Rightarrow$ $a(a^n - b^n) - b(a^n - b^n) = 0$

$\Rightarrow$ $a^n - b^n = 0$ $\Rightarrow$ $\left(\dfrac{a}{b}\right)^n = 1 = \left(\dfrac{a}{b}\right)^0$

$\Rightarrow$ n = 0

74. If α, β, γ and δ are the roots of the equation $x^2(4x^2 - 9) + x(4x - 6) = 6$, then the value of $[\alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1}]$ will be

(a) $-\dfrac{3}{2}$ (b) $-\dfrac{5}{4}$ (c) -1 (d) 0

Sol. **(c)**

$x^2(4x^2 - 9) + x(4x - 6) = 6$

$\Rightarrow$ $4x^4 - 9x^2 + 4x^2 - 6x - 6 = 0$

$\Rightarrow$ $4x^4 - 5x^2 + 6x - 6 = 0$ roots are $\alpha,\beta,\gamma,\delta$

$\alpha + \beta + \gamma + \delta = 0$

$\Sigma\alpha\beta = \dfrac{-5}{4}$

$\Sigma\alpha\beta\gamma = \dfrac{-(-6)}{4} = \dfrac{3}{2}$

$\alpha\beta\gamma\delta = \dfrac{-6}{4} = -\dfrac{3}{2}$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\Sigma\alpha\beta\gamma}{\alpha\beta\gamma\delta} = \frac{\frac{3}{2}}{-\frac{3}{2}} = -1$$

75. Given that $f(mn) = f(m + n)$ for all $m,n \in R$ and if $f(2) = 2009$, the value of $f(2^{2009})$ will be

(a) (2008) (b) $(2008)^{2009}$ (c) (2009) (d) (4018)

Sol. **(c)**

$f(1) = f(1 + 1)$ $\Rightarrow$ $f(1) = f(2)$

$f(2.1)$ $\Rightarrow$ $f(2 + 1)$ $\Rightarrow$ $f(3)$

$f(2)$ $\Rightarrow$ $f(1.1) = f(1 + 1) = f(2)$

$f(2.1) = f(2 + 1) = f(3)$

$f(1) = f(2) = f(3) = f(4) = f(5)$

$\Rightarrow$ $f(2^{2009}) = f(1) = f(2) = 2009$

76. If secA + tanA = a, then (sinA) can be expressed as

(a) $\frac{a-1}{a+1}$ (b) $\frac{a}{a+1}$ (c) $\frac{a-1}{a^2+1}$ (d) $\frac{a^2-1}{a^2+1}$

Sol. **(d)**

$\sec A + \tan A = a$

$\tan A = a - \sec A$

$\tan^2 A = a^2 + \sec^2 A - 2a \sec A$

$2a \sec A = a^2 + 1$

and $\sec A = a - \tan A \Rightarrow \sec^2 A = a^2 + \tan^2 A - 2a \tan A$

$\frac{2a \sin A}{\cos A} = a^2 - 1$

$\frac{2a \sin A}{\cos A} = a^2 - 1$

from (1) $(a^2 + 1) \sin A = a^2 - 1$

$\sin A = \frac{a^2-1}{a^2+1}$

77. If α and β are the roots of the equation $x^2 + x + 3 = 0$, then $(\alpha^6 + \beta^6)$ is

(a) –10 (b) 10 (c) –30 (d) 30

Sol. **(b)**

$x^3 + x + 3 = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = -1,\ \alpha\beta = 3 \Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 1 - 2(3) = -5$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= 25 - 2\times(3)^2 = 7$

$(\alpha^6 + \beta^2) = (\alpha^2)^3 + (\beta^2)^3$

$= (\alpha^2 + \beta^2)(\alpha^4 + \beta^4 - \alpha^2\beta^2)$

$= (-5)(7 - (3)^2)$

$= (-5)(7 - 9)$

$= (-5)(-2)$

$= 10$

78. An electron and a proton are projected in a uniform magnetic field with equal linear momentum in a direction perpendicular to the field. The

(a) the electron trajectory will be less curved than the proton trajectory

(b) the electron trajectory will be more curved than the proton trajectory

(c) both the trajectories will be equally curved

(d) both the particles will have no deflection and will follow a rectilinear trajectory

Sol. **(c)** $r = \frac{mv}{qB} = \frac{p}{qB} \Rightarrow r \propto \frac{1}{q}$

$r_1 : r_2 :: 1 : 1$

79. A pipe of length 40 cm is open at both ends. This pipe is excited by a 1700 Hz source. Given that the speed of sound in air is 340 ms^{-1}, which harmonic mode will be resonantly excited?

(a) First harmonic (b) Second harmonic (c) Third harmonic (d) Fourth harmonic

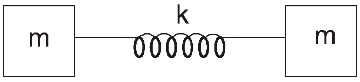
Sol. **(d)** $n\frac{\lambda}{2} = 40$ cm. $\Rightarrow \lambda = \frac{80}{n}$ cm.

$v = f\lambda \Rightarrow 340 = 1700 \times \frac{80}{n} \times 10^{-2}$

$n = 4$

80. Two same masses are connected by a spring satisfying the Hooke's law and are placed on a frictionless table. The spring is elongated along their length a little and allowed to go. Let the angular frequency of oscillations be ω. Now one of the masses is stopped. The square of the new angular frequency is

(a) ω^2 (b) $\omega^2 / 2$ (c) $\omega^2 / 3$ (d) $2\,\omega^2$

Sol. **(b)** 

$$\omega = \sqrt{\frac{K}{\mu}} = \sqrt{\frac{2k}{m}}$$

$$\omega^1 = \sqrt{\frac{K}{m}} = \frac{\omega}{\sqrt{2}}$$