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CODE-A

SUBJECT : MATHEMATICS

WEST BENGAL JOINT ENTRANCE EXAMINATION (WBJEE) 2018

Date: 22 April, 2018 | Duration: 2 Hours | Max. Marks: 100

:: IMPORTANT INSTRUCTIONS ::

1. This question paper contains all objective questions divided into three categories. Each question has four answer options given.
2. **Category-I :** Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer ¼ marks will be deducted.
3. **Category-II :** Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer ½ marks will be deducted.
4. **Category-III :** Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong but there is no negative marking for the same and zero marks will be awarded.
5. Questions must be answered on, OMR sheet by darkening the appropriate bubble marked (A), (B), (C) or (D).
6. Use only Black/Blue ball point pen to mark the answer by complete filing up of the respective bubbles.
7. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
8. Write question booklet number and your roll number carefully in the specified locations of the OMR. Also fill appropriate bubbles.
9. Write your name (in block letter), name of the examination centre and put you full signature in appropriate boxes in the OMR.
10. The OMRs will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination center or signature of the candidate vis-à-vis what is given in the candidate's admit card. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
11. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will reported against & his/her candidature will be summarily cancelled.
12. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
13. Hand over the OMR to the invigilator before leaving the Examination Hall.
14. This paper contains questions in both English and Bengali. Necessary care and precaution were taken while framing the Bengali version. However if any discrepancy(ies) is/are found between the two versions, the information provided in the English version will stand and will be treated as final.

MATHEMATICS

Category – I (Q.1 to Q.50)

Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ marks will be deducted.

1. If $(2 \le r \le n)$, then ${}^nC_r + 2.{}^nC_{r+1} + {}^nC_{r+2}$ is equal to

(A) $2.{}^nC_{r+2}$ (B) ${}^{n+1}C_{r+1}$ (C) ${}^{n+2}C_{r+2}$ (D) ${}^{n+1}C_r$

Ans. **(C)**

Sol. ${}^nC_r + {}^nC_{r+1} + {}^nC_{r+1} + {}^nC_{r+2} = {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2} = {}^{n+2}C_{r+2}$

2. The number $(101)^{100} - 1$ is divisible by

(A) 10^4 (B) 10^6 (C) 10^8 (D) 10^{12}

Ans. **(A)**

Sol. $(101)^{100} - 1$

$= (100 + 1)^{100} - 1$

$= {}^{100}C_0 . 100^{100} + {}^{100}C_1 . 100^{99} + ... + {}^{100}C_{99} . 100 + {}^{100}C_{100} . 1 - 1$

$= 10^4 ({}^{100}C_0 . 100^{96} + ... + 1)$

3. If n is even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may also have the greatest coefficient is

(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (B) $\frac{n}{n+1} < x < \frac{n+1}{n}$ (C) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$ (D) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

Ans. **(A)**

Sol. For greatest term we have

$$\frac{n}{2} < \frac{n+1}{1+|x|} \le \frac{n}{2} + 1$$

$$\Rightarrow \frac{n}{2} < \frac{n+1}{1+x} \text{ and } \frac{n+1}{1+|x|} \le \frac{n}{2} + 1$$

$$\Rightarrow 1 + x < \frac{n+1}{n/2} \text{ and } \frac{n+1}{\frac{n}{2}+1} - 1 \le x$$

$$\Rightarrow x < \frac{n+1-\frac{n}{2}}{\frac{n}{2}} \text{ and } \frac{n+1-\frac{n}{2}-1}{\frac{n+2}{2}} \le x$$

$$\Rightarrow x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} \le x$$

$$\therefore \frac{n}{n+2} < x < \frac{n+2}{n}$$

4. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$, then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

(A) A^2 (B) $A^2 - A + I_3$ (C) $A^2 - 3A + I_3$ (D) $3A^2 + 5A - 4I_3$

I_3 denotes the det of the identity matrix of order 3

Ans. **(A)**

Sol. $A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = -1(1+12) - 7(2+9) = -13 - 77 = -90$

$$B = \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix} = 15\begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix} = 15\begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix} = 15(0 - 0 - 6(-13-77)) = 90 \times 90$$

5. If $a_r = (\cos 2r\pi + i\sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_i & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(A) 1 (B) –1 (C) 0 (D) 2

Ans. **(C)**

Sol. $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, $a_r = e^{\frac{2r\pi\hat{i}}{9}}$

$$= \begin{vmatrix} e^{\frac{2\pi\hat{i}}{9}} & e^{\frac{4\pi\hat{i}}{9}} & e^{\frac{6\pi\hat{i}}{9}} \\ e^{\frac{8\pi\hat{i}}{9}} & e^{\frac{10\pi\hat{i}}{9}} & e^{\frac{12\pi\hat{i}}{9}} \\ e^{\frac{14\pi\hat{i}}{9}} & e^{\frac{16\pi\hat{i}}{9}} & e^{\frac{18\pi\hat{i}}{9}} \end{vmatrix} = e^{\frac{2\pi\hat{i}}{9}} \times e^{\frac{8\pi\hat{i}}{9}} \begin{vmatrix} 1 & e^{\frac{2\pi\hat{i}}{9}} & e^{\frac{4\pi\hat{i}}{9}} \\ 1 & e^{\frac{2\pi\hat{i}}{9}} & e^{\frac{2\pi\hat{i}}{9}} \\ \ldots & \ldots & \ldots \end{vmatrix} = 0$$

6. If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of

(A) x only (B) y only (C) n only (D) x, y, z and n

Ans. **(D)**

Sol. $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$

$$\Rightarrow \sum_{r=1}^{n} S_r = \begin{vmatrix} 2\sum_{r=1}^{n} r & x & n(n+1) \\ \sum_{r=1}^{n}(6r^2 - 1) & y & n^2(2n+3) \\ \sum_{r=1}^{n}(4r^3 - 2nr) & z & n^3(n+1) \end{vmatrix} = \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix} = 0$$

7. If the following three linear equations have a non-trivial solution, then

$x + 4ay + az = 0$
$x + 3by + bz = 0$
$x + 2cy + cz = 0$

(A) a,b,c are in A.P. (B) a,b,c are in G.P.
(C) a,b,c are in H.P. (D) $a + b + c = 0$

Ans. **(C)**

Sol. For non trivial solution

$$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

$\Rightarrow 1(3bc - 2bc) - 1(4ac - 2ac) + (4ab - 3ab) = 0$

$bc - 2ac + ab = 0$

$bc + ab = 2ac$

$b = \dfrac{2ac}{a+c}$ H.P.

8. On R, a relation ρ is defined by $x\rho y$ if and only if $x - y$ is zero or irrational. Then

(A) ρ is equivalence relation
(B) ρ is reflexive but neither symmetric nor transitive
(C) ρ is reflexive & symmetric but not transitive
(D) ρ is symmetric & transitive but not reflexive

Ans. **(C)**

Sol. $xRy \Rightarrow x - y$ is zero or irrational

$xRx \Rightarrow 0 \therefore$ reflective

if $xRy \Rightarrow x - y$ is zero or irrational

$\Rightarrow y - x$ is zero or irrational

$\therefore$ yRx symmetric

$xRy \Rightarrow x - y$ is 0 or irrational

$yRz \Rightarrow y - z$ is 0 or irrational

then $(x - y) + (y - z) = x - z$ may be rational

$\therefore$ it is not transitive

9. On the set R of real numbers, the relation ρ is defined by $x\rho y$, $(x,y) \in R$.

(A) if $|x - y| < 2$ then ρ is reflexive but neither symmetric nor transitive
(B) if $x - y < 2$ then ρ is reflexive and symmetric but not transitive
(C) if $|x| \geq y$ then ρ is reflexive and transitive but not symmetric
(D) if $x > |y|$ then ρ is transitive but neither reflexive nor symmetric

Ans. **(D)**

Sol. $(x, x) \in R \Rightarrow x > |x|$ false

not reflexive

$(x, y) \in R \Rightarrow x > |y| \not\Rightarrow y > |x|$

$\therefore$ not symmetric

$(x, y) \in R \Rightarrow x > |y|, (y, z) \in R \Rightarrow y > |z|$

$\Rightarrow x > |z| \Rightarrow (x, z) \in R$

$\therefore$ Transitive

10. If $f : R \to R$ be defined by $f(x) = e^x$ and $g : R \to R$ be defined by $g(x) = x^2$. The mapping $gof : R \to R$ be defined by $(gof)(x) = g[f(x)]\ \forall\ x \in R$, Then

(A) gof is bijective but f is not injective

(B) gof is injective and g is injective

(C) gof is injective but g is not bijective

(D) gof is surjective and g is surjective

Ans. **(C)**

Sol. $f(x) = e^x : R \to R$

$g(x) = x^2 : R \to R$

$g(f(x)) = g(e^x) = (e^x)^2 = e^{2x}\ \forall\ x \in R$

clearly g(f(x)) is injective and g(x) is not injective

11. In order to get a head at least once with probability ≥ 0.9, the minimum number of times a unbiased coin needs to be tossed is

(A) 3 (B) 4 (C) 5 (D) 6

Ans. **(B)**

Sol. $P(H) = \frac{1}{2}, P(T) = \frac{1}{2}$

$P = 1 - \frac{1}{2^n} \geq 0.9$

$1 - \frac{9}{10} \geq \frac{1}{2^n} \Rightarrow \frac{1}{2^n} \leq \frac{1}{10}$

$\Rightarrow 10 \leq 2^n$

$n = 4$

12. A student appears for tests I, II and III. The student is successful if he passes in tests I, II or I, III. The probabilities of the student passing in tests I, II and III are respectively p, q and $\frac{1}{2}$. If the probability of the student to be successful is $\frac{1}{2}$. Then

(A) $p(1 + q) = 1$ (B) $q(1 + p) = 1$ (C) $pq = 1$ (D) $\frac{1}{p} + \frac{1}{q} = 1$

Ans. **(A)**

Sol. $\frac{1}{2} = P(I)P(II)P(III') + P(I)P(II')P(III) + P(I)P(II)P(III)$

$= p.q.\left(1 - \frac{1}{2}\right) + P.(1 - q)\cdot\frac{1}{2} + p.q\cdot\frac{1}{2}$

$1 = pq + p - pq + pq \Rightarrow 1 = pq + p = p(q + 1)$

13. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$ (C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

(n is integer)

Ans. **(A)**

Sol. $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\sin 4\theta + 2 \sin 4\theta \cos 2\theta = 0$

$\sin 4\theta (1 + 2 \cos 2\theta) = 0$

$\sin 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2} = \cos \frac{2\pi}{3}$

$4\theta = n\pi$ or $2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = \frac{n\pi}{4}$ or $\theta = n\pi \pm \frac{\pi}{3}$

14. If $0 \le A \le \frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ is equal to

(A) $\frac{\pi}{4}$ (B) π (C) 0 (D) $\frac{\pi}{2}$

Ans. **(B)**

Sol. $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot 3A}{1 - \cot^4 A}\right)$

$= \tan^{-1}\left(\frac{1}{2}\cdot\frac{2\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right)$

$= \pi + 0 = \pi$

15. Without changing the direction of the axes, the origin is transferred to the point (2, 3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

(A) $x^2 + y^2 + 4 = 0$ (B) $x^2 + y^2 = 4$

(C) $x^2 + y^2 - 8x - 12y + 48 = 0$ (D) $x^2 + y^2 = 9$

Ans. **(B)**

Sol. $x \to x + 2, y \to y + 3$

$\therefore \quad (x + 2)^2 + (y + 3)^2 - 4(x + 2) - 6(y + 3) + 9 = 0$

$\Rightarrow \quad x^2 + 4x + 4 + y^2 + 6y + 9 - 4x - 8 - 6y - 18 + 9 = 0$

$\Rightarrow \quad x^2 + y^2 - 4 = 0$

16. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is

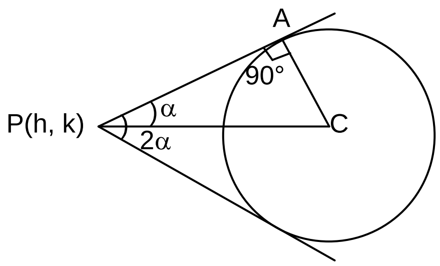
(A) $x^2 + y^2 + 4x + 6y + 9 = 0$ (B) $x^2 + y^2 - 4x + 6y + 9 = 0$

(C) $x^2 + y^2 - 4x - 6y + 9 = 0$ (D) $x^2 + y^2 + 4x - 6y + 9 = 0$

Ans. **(D)**

Sol. $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

C(–2, 3), r = $\sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha} = \sqrt{13\sin^2\alpha - 9\sin^2\alpha} = 2\sin\alpha$



$\sin\alpha = \dfrac{AC}{PC}$

$PC^2\sin^2\alpha = 4\sin^2\alpha$

$(h + 2)^2 + (k - 3)^2 = 4$ $\quad\therefore$ locus of P is $x^2 + y^2 + 4x - 6y + 9 = 0$

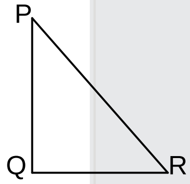
17. The point Q is the image of the point P(1, 5) about the line y = x and R is the image of the point Q about the line y = – x. The circumcenter of the ΔPQR is

(A) (5, 1) (B) (–5, 1) (C) (1, – 5) (D) (0, 0)

Ans. **(D)**

Sol. Clearly P(1, 5)

Q(5, 1) R(–1, –5)



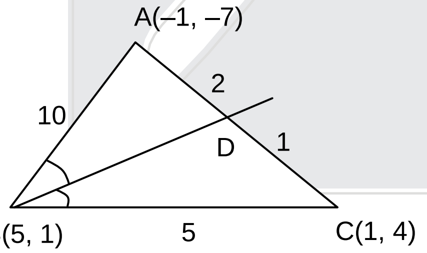
$\therefore$ circumcentre of PQR is $\left(\dfrac{1-1}{2}, \dfrac{5-5}{2}\right) = (0, 0)$

18. The angular points of a triangle are A(–1, –7), B(5, 1) and C(1, 4). The equation of the bisector of the angle ∠ABC is

(A) x = 7y + 2 (B) 7y = x + 2 (C) y = 7x + 2 (D) 7x = y + 2

Ans. **(B)**

Sol.



$AB = \sqrt{36+64} = 10$, $BC = \sqrt{16+9} = 5$

$D\left(\dfrac{-1+2}{3}, \dfrac{-7+8}{3}\right) = D\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$

$\therefore$ Equation of BD is $y - 1 = \dfrac{1-\frac{1}{3}}{5-\frac{1}{3}}(x-5)$

$y - 1 = \dfrac{2}{14}(x - 5)$

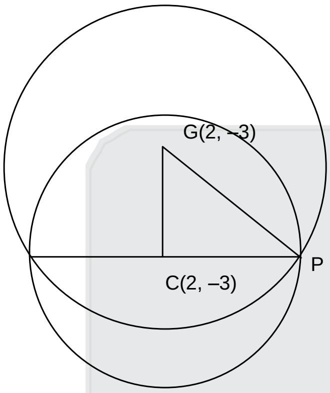
$7y - 7 = x - 5 \;\therefore\; x - 7y + 2 = 0 \;\Rightarrow\; x + 2 = 7y$

19. If one the diameters of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2, –3), the radius of S is

(A) $\sqrt{41}$ unit (B) $3\sqrt{5}$ unit (C) $5\sqrt{2}$ unit (D) $2\sqrt{5}$ unit

Ans. **(A)**

Sol. $x^2 + y^2 + 4x + 6y - 12 = 0$



$C(-2, -3)$, $r = \sqrt{4+9+12} = 5$

$CG = \sqrt{16+0} = 4$

$GP^2 = CG^2 + CP^2$

$= 16 + 25$

$C_iP = \sqrt{41}$

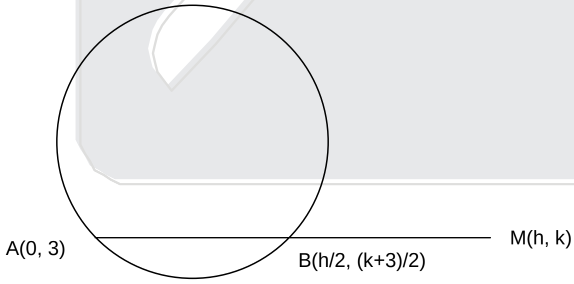
20. A chord AB is drawn from the point A(0, 3) on the circle $x^2 + 4x + (y-3)^2 = 0$, and is extended to M such that AM = 2AB. The locus of M is

(A) $x^2 + y^2 - 8x - 6y + 9 = 0$ (B) $x^2 + y^2 + 8x + 6y + 9 = 0$

(C) $x^2 + y^2 + 8x - 6y + 9 = 0$ (D) $x^2 + y^2 - 8x + 6y + 9 = 0$

Ans. **(C)**

Sol.



$$\frac{h^2}{4} + 2h + \left(\frac{k+3}{2} - 3\right)^2 = 0$$

$$\frac{h^2}{4} + 2h + \frac{k^2 - 6k + 9}{4} = 0$$

$\therefore$ locus of M is $x^2 + y^2 + 8x - 6y + 9 = 0$

21. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

(A) 8 : 1 (B) 1 : 8 (C) 9 : 1 (D) 1 : 9

Ans. **(A)**

Sol. Eccentricity of ellipse e = $\sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}}$

eccentricity of hyperbola = $\sqrt{\frac{9}{8}}$

$1+\frac{b^2}{a^2}=\frac{9}{8}$

$\frac{b^2}{a^2}=\frac{1}{8}$

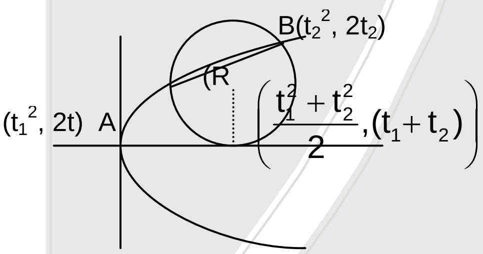
$a^2 : b^2 = 8 : 1$ Ans. (A)

22. Let A,B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Ans. **(CD)**

Sol.



radius of circle = $|t_1 + t_2| = r$

slope of line AB = $\frac{2}{t_1+t_2} = \frac{2}{\pm r}$ Ans. (C, D)

23. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the co-ordinates of K is (2a, 0), then the value of r is

(A) $\frac{t}{1-t^2}$ (B) $\frac{1-t^2}{t}$ (C) $\frac{t^2+1}{t}$ (D) $\frac{t^2-1}{t}$

Ans. **(D)**

Sol. $m_{PK} = m_{QR}$

$\frac{2at-0}{at^2-2a}=\frac{2at'-2ar}{a(t')^2-ar^2}$

$\frac{t}{t^2-2}=\frac{t'-r}{(t')^2-r^2}$

$-t'-tr^2=-t-rt^2-2t'+2r\ ,\ tt'=-1$

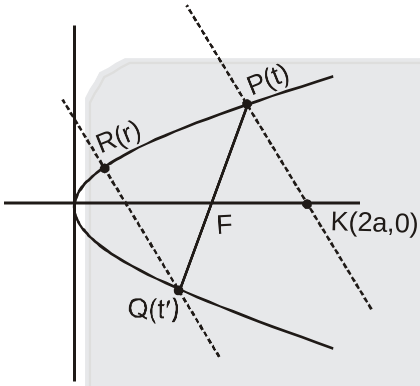
$t'-tr^2=-t+2r-rt^2$

$-tr^2+r(t^2-2)+t'+t=0$

$$\lambda = \frac{(2-t^2) \pm \sqrt{(t^2-2)^2 + 4(-1+t^2)}}{-2t} = \frac{(2-t^2) \pm \sqrt{t^4}}{-2t} = \frac{2-t^2 \pm t^2}{-2t}$$

$r = -\frac{1}{t}$ It is not possible as the R & Q will be one same.

or $r = \frac{t^2-1}{t}$ (D) Ans.



24. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line through P parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P, Q are on the same side of the x-axis. If R is a point on PQ such that $\frac{PR}{RQ} = \frac{1}{2}$, then the locus of R is

(A) $\frac{x^2}{9} + \frac{9y^2}{49} = 1$ (B) $\frac{x^2}{49} + \frac{y^2}{9} = 1$ (C) $\frac{x^2}{9} + \frac{y^2}{49} = 1$ (D) $\frac{9x^2}{49} + \frac{y^2}{49} = 1$

Ans. **(A)**

Sol. $P(3\cos\theta, 2\sin\theta)$

$Q(3\cos\theta, 3\sin\theta)$

1 2

P R(h,k) Q

$(3\cos\theta, 2\sin\theta)$ $(3\cos\theta, 3\sin\theta)$

$h = \frac{3\cos\theta + 6\cos\theta}{3}$, $k = \frac{3\sin\theta + 4\sin\theta}{3}$

$h = 3\cos\theta$, $k = \frac{7}{3}\sin\theta$.

$\therefore$ $\sin^2\theta + \cos^2\theta = 1$

$\frac{h^2}{9} + \frac{9k^2}{49} = 1$

locus is $\frac{x^2}{9} + \frac{9y^2}{49} = 1$

25. A point P lies on a line through Q(1, –2, 3) and is parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$. If P lies on the plane $2x + 3y – 4z + 22 = 0$, then segment PQ equals to

(A) $\sqrt{42}$ units (B) $\sqrt{32}$ units (C) 4 unit (D) 5 units

Ans. **(A)**

Sol. Equation line $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$

$P(\lambda + 1, 4\lambda – 2, 5\lambda + 3)$

P lies on $2x + 3y – 4z + 22 = 0$

$2(\lambda + 1) + 3(4\lambda – 2) – 4(5\lambda + 3) + 22 = 0$

$–6\lambda + 6 = 0$

$\lambda = 1$

P(2, 2, 8)

$PQ = \sqrt{1+16+25}=\sqrt{42}$ Ans. (A)

26. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, –11, 4) and (2, – 3, 1) is

(A) (4, 5, 2) (B) (–4, 5, 2) (C) (4, –5, 2) (D) (4, 5, –2)

Ans. **(D)**

Sol. Equation of line joining points (0, –11, 4) and (2, –3, 1)

$\frac{x-2}{2}=\frac{y+3}{8}=\frac{z-1}{-3}=\lambda$

DR's of PQ $2\lambda + 1, 8\lambda – 11, –3\lambda – 3$

Now $(2\lambda + 1)2 + (8\lambda – 11)8 + (–3\lambda – 3)(–3) = 0$

$77\lambda – 77 = 0 \quad \Rightarrow \quad \lambda = 1$

Q(4, 5, –2) Ans. (D)

27. The approximate value of sin31º is

(A) > 0.5 (B) > 0.6 (C) < 0.5 (D) < 0.4

Ans. **(A)**

Sol. $\therefore \sin 30° = \frac{1}{2}$

$\therefore$ sinx is increasing function $\Rightarrow \sin 31° > \frac{1}{2}$

28. Let $f_1(x) = e^x$, $f_2(x) = e^{f_1(x)}$,...., $f_{n+1}(x) = e^{f_n(x)}$ for all $n \ge 1$. The for any fixed n, $\frac{d}{dx}f_n(x)$ is

(A) $f_n(x)$ (B) $f_n(x)\, f_{n-1}(x)$ (C) $f_n(x)\, f_{n-1}(x)...f_1(x)$ (D) $f_n(x).....f_1(x)\, e^x$

Ans. **(C)**

Sol. $\frac{d}{dx}f_n(x) = f_n(x) \,.\, f_{n-1}(x) .... f_1(x)$

29. The domain of definition of $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$ is

(A) $(-\infty, -1) \cup (2, \infty)$
(B) $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$
(C) $(-\infty, 1) \cup (2, \infty)$
(D) $[-1, 1] \cup (2, \infty)$

Here $(a, b) \equiv \{x : a < x < b\}$ & $[a, b] \equiv \{x : a \le x \le b\}$

Ans. **(B)**

Sol. $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$

$\dfrac{1-|x|}{2-|x|} \ge 0 \Rightarrow |x| \le 1$ or $|x| > 2 \Rightarrow x \in [-1, 1]$ or $x \in (-\infty, -2) \cup (2, \infty)$ Ans. (B)

30. Let $f : [a, b] \to R$ be differentiable on $[a, b]$ and $k \in R$. Let $f(a) = 0 = f(b)$. Also let $J(x) = f'(x) + kf(x)$. Then

(A) $J(x) > 0$ for all $x \in [a, b]$
(B) $J(x) < 0$ for all $x \in [a, b]$
(C) $J(x) = 0$ has atleast one root in (a, b)
(D) $J(x) = 0$ through (a, b)

Ans. **(C)**

Sol. Let $g(x) = e^{kx} f(x)$

$f(a) = 0 = f(b)$

by rolles theorem

$g'(c) = 0, \ c \in (a, b)$

$g'(x) = e^{kx} f'(x) + ke^{kx} f(x)$

$g'(c) = 0$

$e^{kc}(f'(c) + kf(c)) = 0$

$\Rightarrow f'(c) + kf(c) = 0$ for atleast one c in (a, b)

Ans. C

31. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\dfrac{f(1-h) - f(1)}{h^3 + 3h}$

(A) does not exist
(B) is $\dfrac{50}{3}$
(C) is $\dfrac{53}{3}$
(D) is $\dfrac{22}{3}$

Ans. **(C)**

Sol. $\lim\limits_{h \to 0} \dfrac{f(1-h) - f(1)}{h^3 + 3h} \quad \left(\dfrac{0}{0} \text{form}\right)$

$\lim\limits_{h \to 0} \dfrac{-f'(1-h)}{3h^2 + 3} = \dfrac{-f'(1)}{3}$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$f'(1) = 30 - 56 + 30 - 63 + 6 = -53$ Ans. (C)

32. Let $f : [a, b] \to R$ be such that f is differentiable in (a, b), f is continuous at $x = a$ and $x = b$ and moreover $f(a) = 0 = f(b)$. Then

(A) there exists atleast one point c in (a, b) such that $f'(c) = f(c)$
(B) $f'(x) = f(x)$ does not hold at any point in (a, b)
(C) at every point of (a, b), $f'(x) > f(x)$
(D) at every point of (a, b), $f'(x) < f(x)$

Ans. **(A)**

Sol. Let $h(x) = e^{-x} f(x)$

$h(a) = 0, \ h(b) = 0$

h(x) is continuous and diff.

by rolles theorem

$h'(c) = 0,\ c \in (a, b)$

$e^{-x}f(x) + (-e^{-x})f(x) = 0$

$e^{-c} f'(c) = e^{-c}f(c)$

$f'(c) = f(c)$

33. Let $f : R \to R$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$. Then

(A) $f''(0) = 0$
(B) $f''(c) = 0$ for some $c \in R$
(C) if $c \neq 0$, then $f''(c) \neq 0$
(D) $f'(x) > 0$ for all $x \neq 0$

Ans. **(B)**

Sol. f(x) is continuous and differentiable
$f(0) = f(1) = 0 \Rightarrow$ by rolles theorem
$f'(a) = 0,\ a \in (0, 1)$
given $f'(0) = 0$
by rolles theorem $f''(0) = 0$ for some c, $c \in (0, a)$
Ans. B

34. If $\int e^{\sin x}\left[\frac{x\cos^3 x - \sin x}{\cos^2 x}\right]dx = e^{\sin x}f(x) + c$ where c is constant of integration, then f(x) =

(A) $\sec x - x$
(B) $x - \sec x$
(C) $\tan x - x$
(D) $x - \tan x$

Ans. **(B)**

Sol. $\int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right)dx$

$= \int e^{\sin x}(x\cos x - \tan x\sec x)dx$

$= (xe^{\sin x} - \int e^{\sin x}) - \left[e^{\sin x}\sec x - \int e^{\sin x}dx\right] + c$

$= e^{\sin x}(x - \sec x) + c$

Ans. B

35. If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2 - a^2)}\log f(x) + c$, where c is the constant of integration, then f(x) =

(A) $\frac{2}{(b^2 - a^2)\sin 2x}$
(B) $\frac{2}{ab\sin 2x}$
(C) $\frac{2}{(b^2 - a^2)\cos 2x}$
(D) $\frac{2}{ab\cos 2x}$

Ans. **(C)**

Sol. check by option

36. If $M = \int_0^{\pi/2}\frac{\cos x}{x+2}dx,\ N = \int_0^{\pi/4}\frac{\sin x\cos x}{(x+1)^2}dx$, then the value of $M - N$ is

(A) π
(B) $\frac{\pi}{4}$
(C) $\frac{2}{\pi - 4}$
(D) $\frac{2}{\pi + 4}$

Ans. **(D)**

Sol. $N = \int_0^{\pi/4}\frac{\sin 2x}{2(x+1)^2}dx$

Let $2x = t$

$2dx = dt$

$$N = \int_0^{\pi/2} \frac{\sin t}{4\left(\frac{t}{2}+1\right)^2} dt = \int_0^{\pi/2} \frac{\sin t}{(t+2)^2} dt$$

$$= \sin t \left(-\frac{1}{t+2}\right) + \int_0^{\pi/2} \frac{\cos t}{t+2} dt$$

$$N = \left(-\sin t\left(\frac{1}{t+2}\right)\right)_0^{\pi/2} + M$$

$$M - N = \frac{2}{\pi+4} \text{ Ans. D}$$

37. The value of the integral $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$ is

(A) $\frac{\pi}{4}$ log 2014 (B) $\frac{\pi}{2}$ log 2014 (C) π log 2014 (D) $\frac{1}{2}$ log 2014

Ans. **(B)**

Sol. $I = \int_{\frac{1}{2014}}^{2014} \frac{\tan^{-1} x}{x} dx$ ....(1)

Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$$I = \int_{2014}^{1/2014} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\left(\frac{1}{t}\right)}\left(-\frac{1}{t^2}\right) dt$$

$$I = \int_{1/2014}^{2014} \frac{\cot^{-1} t}{t} dt \quad ....(2)$$

from (1) + (2)

$$2I = \int_{1/2014}^{2014} \frac{\pi/2}{t} dt$$

$$I = \frac{\pi}{4}(\ell n t)_{1/2014}^{2014}$$

$$= \frac{\pi}{4}\left(\ell n 2014 - \log \frac{1}{2014}\right)$$

$$= \frac{\pi}{4}(2\,\ell n\, 2014) = \frac{\pi}{2}\,\ell n\, 2014$$

38. Let $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$. Then

(A) $\frac{1}{2} \le I \le 1$ (B) $4 \le I \le 2\sqrt{30}$ (C) $\frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$ (D) $1 \le I \le \frac{2\sqrt{3}}{\sqrt{2}}$

Ans. **(C)**

Sol. $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$

$\frac{\sin x}{x}$ is a decreasing function

so $\frac{\pi}{12} \times \frac{\sin \pi/3}{\pi/3} \le I \le \frac{\pi}{12} \times \frac{\sin \pi/4}{\pi/4}$

$\frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$

Ans. C

39. The value of $I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$, is

(A) 1 (B) π (C) e (D) $\pi/2$

Ans. **(B)**

Sol. $I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$

$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$ ...... (1)

$I = \int_{\pi/2}^{\pi} \frac{e^{-\tan^{-1}(\sin x)}}{e^{-\tan^{-1}(\sin x)} + e^{-\tan^{-1}(\cos x)}} dx + \int_{\pi}^{5\pi/2} \frac{e^{-\tan^{-1}(\sin x)}}{e^{-\tan^{-1}(\sin x)} + e^{-\tan^{-1}(\cos x)}}$ by $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{5\pi/2} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$ ...... (2)

from (1) + (2)

$2I = \int_{\pi/2}^{\pi} 1\,dx + \int_{\pi}^{5\pi/2} 1\,dx$

$2I = (x)_{\pi/2}^{5\pi/2} = 2\pi$

$I = \pi$

40. The value of

$$\lim_{n\to\infty}\frac{1}{n}\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+........+\sec^2\frac{n\pi}{4n}\right\} \text{ is}$$

(A) $\log_e 2$ (B) $\frac{\pi}{2}$ (C) $\frac{4}{\pi}$ (D) e

Ans. **(C)**

Sol. $\int_0^1 \sec^2\frac{\pi x}{4}dx$

$$\left(\frac{\tan\frac{\pi x}{4}}{\frac{\pi}{4}}\right)_0^1 = \frac{4}{\pi}$$

41. The differential equation representing the family of curves $y^2 = 2d(x+\sqrt{d})$ where d is a parameter, is of

(A) order 2 (B) degree 2 (C) degree 3 (D) degree 4

Ans. **(C)**

Sol. $2y\frac{dy}{dx} = 2d \quad\Rightarrow\quad d = \frac{ydy}{dx}$

$$y^2 = 2y\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)$$

$$y^2 = 2y\frac{dy}{dx} + 2y^{3/2}\left(\frac{dy}{dx}\right)^{3/2}$$

$$(y^2 - 2xy\frac{dy}{dx})^2 = 4y^3\left(\frac{dy}{dx}\right)^3$$

Degree three

42. Let y(x) be a solution of $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and $y(0) = -1$. Then y(1) is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) –1

Ans. **(C)**

Sol. I.F. = $(1+x^2)$

$y(1+x^2) = \int 4x^2dx \quad\Rightarrow\quad y(1+x^2) = \frac{4x^3}{3} - 1 \quad$ (as $y(0) = -1$)

$y(1) = \frac{1}{6}$

43. The law of motion of a body moving along a straight line is $x = \frac{1}{2}vt$, x being its distance from a fixed point on the line at time t and v is its velocity there. Then

(A) acceleration f varies directly with x
(B) acceleration f varies inversely with x
(C) acceleration f is constant
(D) acceleration f varies directly with t

Ans. **(C)**

Sol. $x = \frac{1}{2}vt$

$x = \frac{1}{2} \cdot \frac{dx}{dt} \cdot t$

$\frac{2dt}{t} = \frac{dx}{x}$

$\ell nc + 2\ell nt = \ell nx$

$x = t^2c$

$\frac{dx}{dt} = 2tc$

$\frac{d^2x}{dt^2} = 2c$

Hence acceleration is constant

44. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is

(A) 1
(B) 2
(C) 3
(D) 4

Ans. **(B)**

Sol. $y = x^2$; $y = -(x-2)^2$

$\frac{\alpha^2 + (\beta-2)^2}{\alpha - \beta} = 2\alpha = -2(\beta - 2)$

$\Rightarrow \quad \alpha = 2 - \beta \Rightarrow \beta = 2 - \alpha$

$\frac{\alpha^2 + \alpha^2}{\alpha - 2 + \alpha} = 2a \Rightarrow \frac{2\alpha^2}{2\alpha - 2} = 2\alpha$

$\alpha^2 = \alpha(2\alpha - 2)$

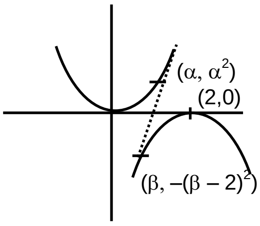
$\Rightarrow \quad \alpha^2 = 2\alpha^2 - 2\alpha$

$\alpha^2 = 2\alpha \Rightarrow \alpha = 0, 2$

$\alpha = 0 \quad \beta = 2$

$\alpha = 2 \quad \beta = 0$

Hence two common tangent



45. Given that n numbers of A.Ms are inserted between two sets of numbers a, 2b and 2a, b where a, b $\in$ R. Suppose further that the m^{th} means between these sets of numbers are same, then the ratio a : b equals

(A) $n - m + 1 : m$ (B) $n - m + 1 : n$ (C) $n : n - m + 1$ (D) $m : n - m + 1$

Ans. **(D)**

Sol. a.............n A.M's 2b

$d = \frac{2b - a}{n + 1}$

$A_m = a + m\left(\frac{2b - a}{n + 1}\right)$ ....(1)

2a....................n A.M's ....b

$d = \frac{b - 2a}{n + 1}$

$A_m = 2a + m\left(\frac{b - 2a}{n + 1}\right)$ ....(2)

equating (1) & (2)

$a = \frac{m}{n + 1}(b + a)$

$\Rightarrow \frac{a}{b} = \frac{m}{n - m + 1}$

46. If $x + \log_{10}(1 + 2^x) = x \log_{10}5 + \log_{10}6$ then the value of x is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 1 (D) 2

Ans. **(C)**

Sol. $\log_{10}(1 + 2^x) = \log_{10}5^x + \log_{10}6 - \log_{10}10^x$

$1 + 2^x = \frac{6.5^x}{10^x} = \frac{6}{2^x}$

$t(1 + t) = 6 \Rightarrow t^2 + t - 6 = 0$

$(t + 3)(t - 2) = 0$

$2^x = 2 \Rightarrow x = 1$

47. If $Z_r = \sin\dfrac{2\pi r}{11} - i\cos\dfrac{2\pi r}{11}$ then $\sum\limits_{r=0}^{10} Z_r =$

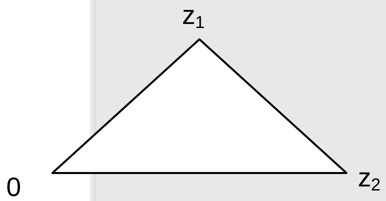
(A) –1 (B) 0 (C) i (D) –i

Sol. $-i\sum\limits_{r=0}^{10}\cos\dfrac{2r\pi}{11} + i\sin\dfrac{2r\pi}{11} \Rightarrow -i\left[\sum\limits_{r=0}^{10} e^{\frac{i2r\pi}{11}}\right] = -i\,[0] = 0$

48. If z_1 and z_2 be two non zero complex numbers such that $\dfrac{z_1}{z_2} + \dfrac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2

(A) lie on a straight line
(B) form a right angled triangle
(C) form an equilateral triangle
(D) form an isosceles triangle

Sol.



$z_1 = z_2 e^{i\pi/3}$

$2z_1 = z_2(1 + i\sqrt{3})$

$2z_1 - z_2 = i\sqrt{3}z_2$

$\Rightarrow 4z_1^2 + z_2^2 - 4z_1z_2 = -3z_2^2 \Rightarrow 4z_1^2 + 4z_2^2 = 4z_1z_2$ Hence form equilateral triangle.

49. If $b_1b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ has

(A) real roots
(B) purely imaginary roots
(C) roots of the form $a + ib$ ($a, b \in R$, $ab \neq 0$)
(D) rational roots

Sol. $D_1 + D_2 = (b_1 - b_2)^2 \geq 0$

Hence real roots

50. The number of selection of n objects from 2n objects of which n are identical and the rest are different is

(A) 2^n (B) 2^{n-1} (C) $2^n - 1$ (D) $2^{n-1} + 1$

Sol. ${}^nC_0 + {}^nC_1 + {}^nC_2 \ldots\ldots + {}^nC_n$

$= 2^n$

Category – II (Q.51 to Q.65)

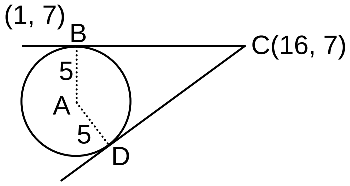
Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer ½ marks will be deducted.

51. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let B(1, 7) and D(4, – 2) be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral ABCD is

(A) 150 sq. units (B) 50 sq. units (C) 75 sq. units (D) 70 sq. units

Ans. **(C)**

Sol.



tangent at B, $y = 7$
tangent at D, $x = 16$

so $BC = 15 \Rightarrow$ area o quadilateral $= 2 \times \frac{1}{2} \times 5 \times 15 = 75$

52. Let $f(x) = \begin{cases} -2\sin x, & \text{if } x \le -\frac{\pi}{2} \\ A\sin x + B, & \text{If } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \ge \frac{\pi}{2} \end{cases}$. Then

(A) f is discontinuous for all A and B

(B) f is continuous for all A = – 1 and B = 1

(C) f is continuous for all A = 1 and B = –1

(D) f is continuous for all real values of A, B

Ans. **(B)**

Sol. for continuity at $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$

$= -A + B$ and $A + B = 0$
$A = -1$ and $B = 1$

53. The normals to the curve $y = x^2 - x + 1$, drawn at the points with the abscissa $x_1 = 0$, $x_2 = -1$ and $x_3 = \frac{5}{2}$

(A) are parallel to each other

(B) are pair wise perpendicular

(C) are concurrent

(D) are not concurrent

Ans. **(C)**

Sol. $\frac{dy}{dx} = 2x - 1, \quad m_N = \frac{1}{1-2x}$

$m_{x_1} = 1$ point (0, 1)

$y - 1 = 1(x)$, $x - y + 1 = 0$ ....(1)

$m_{x_2} = \frac{1}{3}$ point (–1, 3)

$y - 3 = \frac{1}{3}(x + 1) \Rightarrow 3y - 9 = x + 1$

$x - 3y + 10 = 0$ ....(2)

$x_3 = \frac{5}{2}$, $\left(\frac{5}{2}, \frac{19}{4}\right)$

$m_{x_3} = -\frac{1}{4}$

$$y - \frac{19}{4} = -\frac{1}{4}\left(x - \frac{5}{2}\right)$$

$$\Rightarrow\ x + 4y = \frac{43}{2}\ \ ....(3)$$

Intersection point of (1) & (2) $\left(\frac{7}{2}, \frac{9}{12}\right)$ passes (3)

Hence normal are concurrent.

54. The equation x log x = 3 – x

(A) has no root in (1, 3)

(B) has exactly one root in (1, 3)

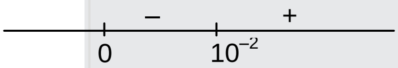
(C) x log x – (3 – x) > 0 in [1, 3]

(D) x log x – (3 – x) < 0 in [1, 3]

Ans. **(B)**

Sol. $f(x) = x\log x - 3 + x$

$f'(x) = 1 + \log x + 1$



$f(1)\,f(3) = -2(3\log 3) = -ve$

Hence one roots in (1, 3)

55. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola where P(4, –4) & Q(9, 6). Let R be a point on the arc of the parabola between P & Q. Then the area of ΔPQR is largest when

(A) $\angle PQR = 90°$

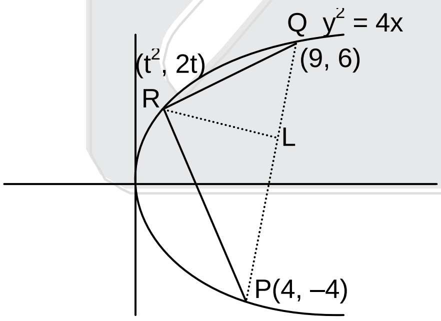
(B) R(4, 4)

(C) R $\left(\frac{1}{4}, 1\right)$

(D) R $\left(1, \frac{1}{4}\right)$

Ans. **(C)**

Sol.



$$PQ \equiv 2x - y = 12$$

$$\perp \text{ distance } RL^2 = \frac{(2t^2 - 2t - 12)^2}{5}$$

$$\text{for maximum } t = \frac{2}{2 \times 2} = \frac{1}{2}$$

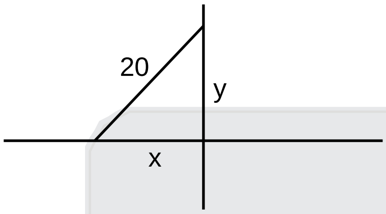
$$R \equiv \left(\frac{1}{4}, 1\right)$$

56. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

(A) $\frac{8}{3}$ (B) $\frac{6}{5}$ (C) $\frac{3}{2}$ (D) $\frac{17}{4}$

Ans. **(A)**

Sol.



$x^2 + y^2 = 400$ given $\frac{dy}{dt} = 2$ ft/sec

$x = 12 \quad \Rightarrow \quad y = 16$

$x\frac{dx}{dt} = -y\frac{dy}{dt}$

$12\left(\frac{dx}{dt}\right) = -16 \times 2$

$\frac{dx}{dt} = -\frac{8}{3}$

57. For $0 \le p \le 1$ and for any positive a, b; let $I(p) = (a + b)^p$, $J(p) = a^p + b^p$, then

(A) $I(p) > J(p)$

(B) $I(p) \le J(p)$

(C) $I(p) < J(p)$ in $\left[0, \frac{p}{2}\right]$ & $I(p) > J(p)$ in $\left[\frac{p}{2}, \infty\right)$

(D) $I(p) < J(p)$ in $\left[\frac{p}{2}, \infty\right)$ & $J(p) < I(p)$ in $\left[0, \frac{p}{2}\right]$

Ans. **(B)**

Sol. $a = 9, b = 16$

$I(P) = 5$ and $J(P) = 7$

$J(P) > I(P)$

Now $a = \frac{1}{9}$ and $b = \frac{1}{16}$

$I(P) = \frac{5}{12}$ $J(P) = \frac{7}{12}$

$J(P) > I(P)$)

58. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$, $\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, whose projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $-\hat{i} - 3\hat{j} - 3\hat{k}$ (B) $\hat{i} - 3\hat{j} - 3\hat{k}$ (C) $-\hat{i} + 3\hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Ans. **(ABC)**

Sol. $\vec{\delta} = \vec{\alpha} + n\vec{\beta} = \left(\hat{i} + \hat{j} + \hat{k}\right) + \left(\hat{i} - \hat{j} - \hat{k}\right)$

$\vec{\delta} = (1+n)\hat{i} + (1-n)\hat{j} + (1-n)\hat{k}$

$\frac{\vec{\delta}.\vec{\gamma}}{|\vec{\gamma}|} = \frac{1}{\sqrt{3}} \Rightarrow \frac{|-1-n+1-n-1+n|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$|n + 1| = 1 \quad \Rightarrow n + 1 \Rightarrow n = 0$

or $\quad n = -2$

$\vec{\delta} = -\hat{i} + 3\hat{j} + 3\hat{k}$

59. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three unit vectors such that $\vec{\alpha}.\vec{\beta} = \vec{\alpha}.\vec{\gamma}$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30°. Then $\vec{\alpha}$ is

(A) $2(\vec{\beta} \times \vec{\gamma})$ (B) $-2(\vec{\beta} \times \vec{\gamma})$ (C) $\pm 2(\vec{\beta} \times \vec{\gamma})$ (D) $(\vec{\beta} \times \vec{\gamma})$

Ans. **(C)**

Sol. $\vec{\alpha} = n\left(\vec{\beta} \times \vec{\gamma}\right)$

$1 = |n| \times 1 \times 1 \times \sin 30° \quad \Rightarrow n = \pm 2$

60. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $Re(z_1) > 0$ and $Im(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

(A) one (B) real and positive (C) real and negative (D) purely imaginary

Ans. **(D)**

Sol. $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$R_e(z_1) > 0 \Rightarrow x_1 > 0$ and $I_m(z_2) < 0 \quad \Rightarrow y_2 < 0$

$|z_1| = |z_2| \Rightarrow |z_1|^2 = |z_2|^2 \quad \Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2$

Now $\left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \overline{\left(\frac{z_1 + z_2}{z_1 - z_2}\right)}$

$$\left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2}\right) = \frac{z_1\bar{z}_1 + z_2\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_2 + z_1\bar{z}_1 + z_1\bar{z}_2 - z_2\bar{z}_1 - z_2\bar{z}_2}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}$$

$$= \frac{2(|z_1|^2 - |z_2|^2)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)} = 0 \quad (\because |z_1|^2 = |z_2|^2)$$

$\Rightarrow \frac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary

61. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

(A) 284×17 (B) 285×17 (C) 284×16 (D) 285×16

Ans. **(A)**

Sol. 1, 2, 3, 4, ... 20

there are 17 way for four consecutive number

number ways = ${}^{20}C_4 - 17$

$= 285 \times 17 - 17$

$= 284 \times 17$

62. The least positive integer n such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is an identity matrix of order 2 is

(A) 4 (B) 8 (C) 12 (D) 16

Ans. **(B)**

Sol. $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

Let A = $\begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix} \Rightarrow$ Where $\lambda = \frac{1}{\sqrt{2}}$

$A^2 = \begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix}\begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix} = \begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix} = \begin{bmatrix} -4\lambda^4 & 0 \\ 0 & -4\lambda^4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans. (B)

63. Let ρ be a relation defined on N, the set of natural numbers, as

$\rho = \{(x, y) \in N \times N : 2x + y = 41\}$ Then

(A) ρ is an equivalence relation (B) ρ is only reflexive relation

(C) ρ is only symmetric relation (D) ρ is not transitive

Ans. **(D)**

Sol. $\rho = \{(x, y) \in N \times N, 2x + y = 41\}$

for reflexive relation $x R x \Rightarrow 2x + x = 41 \Rightarrow x = \frac{41}{3} \notin N$

for symmetric $\Rightarrow x R y \quad \Rightarrow 2x + y = 41 \neq y R x$ (Not symmetric)

for transitive $x R y \Rightarrow 2x + y = 41$ and $y R z \Rightarrow 2y + z = 41$, $x \not R z$ (not transitive) Ans. (D)

64. If the polynomial f(x) = $\begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$, then the constant term of f(x) is

(A) $2 - 3.2^b + 2^{3b}$ (B) $2 + 3.2^b + 2^{3b}$ (C) $2 + 3.2^b - 2^{3b}$ (D) $2 - 3.2^b - 2^{3b}$

Ans. **(A)**

Sol. For constant term [put x = 0]

$$f(x) = \begin{vmatrix} 1 & 2^b & 1 \\ 1 & 1 & 2^b \\ 2^b & 1 & 1 \end{vmatrix} = 1(1-2^b) - 2^b(1-2^{2b}) + 1\,(1-2^b)$$

$$= 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b = 2 - 3.2^b + 2^{3b}$$

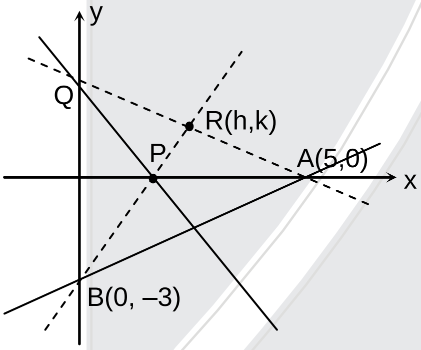
65. A line cuts the x-axis at A(5, 0) and the y-axis at B(0, –3). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P and the y-axis at Q. If AQ and BP meet at R, then the locus of R is

(A) $x^2 + y^2 - 5x + 3y = 0$ (B) $x^2 + y^2 + 5x + 3y = 0$

(C) $x^2 + y^2 + 5x - 3y = 0$ (D) $x^2 + y^2 - 5x - 3y = 0$

Ans. **(A)**

Sol.



Line AB is $\frac{x}{5} + \frac{y}{-3} = 1 \Rightarrow 3x - 5y = 15$

Any perpendicular line to AB

$5x + 3y = \lambda$ So $P\left(\frac{\lambda}{5}, 0\right)$, $Q\left(0, \frac{\lambda}{3}\right)$

AQ is $\frac{x}{5} + \frac{y}{\lambda/3} = 1 \Rightarrow \frac{3y}{\lambda} = 1 - \frac{x}{5} \Rightarrow \frac{1}{\lambda} = \frac{1}{3y}\left(1 - \frac{x}{5}\right)$ .... (1)

and BP is $\frac{x}{\lambda/5} - \frac{y}{3} = 1 \Rightarrow \frac{5x}{\lambda} = 1 + \frac{y}{3} \Rightarrow \frac{1}{\lambda} = \frac{1}{5x}\left(1 + \frac{y}{3}\right)$ .... (2)

$$\frac{1}{3y}\left(1 - \frac{x}{5}\right) = \frac{1}{5x}\left(1 + \frac{y}{3}\right)$$

$$\Rightarrow 5x\left(1 - \frac{x}{5}\right) = 3y\left(1 + \frac{y}{3}\right) \Rightarrow 5x - x^2 = 3y + y^2 \Rightarrow x^2 + y^2 - 5x + 3y = 0$$

Category – III (Q.66 to Q.75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong but there is no negative marking for the same and zero marks will be awarded.

66. In a third order matrix A, a_{ij} denotes the element in the i-th row and j-th column.

If $a_{ij} = 0$ for $i = j$

$= 1$ for $i > j$

$= -1$ for $i < j$

Then the matrix is

(A) skew symmetric (B) symmetric (C) not invertible (D) non-singular

Ans. **(AC)**

Sol. $A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ skew symmetric

$|A| = \begin{vmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 0 + 1(0+1) - 1(1-0)$

$= 1 - 1 = 0$

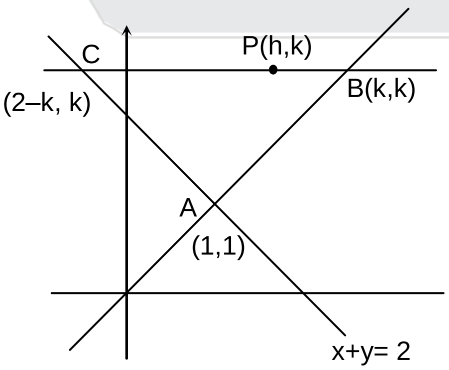
$|A| = 0 \Rightarrow$ non invertible

67. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k), with the lines $y = x$ and $x + y = 2$ is h^2. The locus of the point P is

(A) $x = y - 1$ (B) $x = -(y - 1)$ (C) $x = 1 + y$ (D) $x = -(1 + y)$

Ans. **(AB)**

Sol.



$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & k & k \\ 1 & 2-k & k \end{vmatrix} = \pm h^2$$

$\Rightarrow 1(k^2 - k(2-k)) - 1(k-k) + 1(2-k-k) = \pm 2h^2$

$\Rightarrow k^2 - 2k + k^2 + 2 - 2k = \pm 2h^2$

$\Rightarrow 2k^2 - 4k + 2 = \pm 2h^2$

locus is $(k-1)^2 = h^2 \Rightarrow y - 1 = \pm x$

$x - y + 1 = 0$ or $x + y = 1$

$x = y - 1$ $x = -(y-1)$

68. A hyperbola, having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

(A) $x^2 \sin^2\theta - y^2\cos^2\theta = 1$

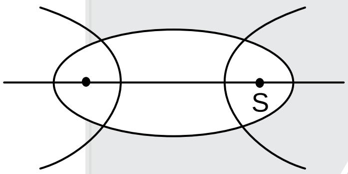
(B) $x^2 \operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

(C) $(x^2 + y^2)\sin^2\theta = 1 + y^2$

(D) $x^2\operatorname{cosec}^2\theta = x^2 + y^2 + \sin^2\theta$

Ans. **(B)**

Sol.



$2A = 2\sin\theta$

$A = \sin\theta$

$3x^2 + 4y^2 = 12$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$, $(a = 2, b = \sqrt{3})$

$\Rightarrow b^2 = a^2(1 - e^2)$

$3 = 4(1 - e^2)$

$\Rightarrow e^2 = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

$S(ae, 0) \Rightarrow S(1, 0)$

for hyperbola foci are same

$Ae_1 = ae = 1$

$\Rightarrow (\sin\theta)e_1 = 1 \Rightarrow e_1 = \operatorname{cosec}\theta$

and $B^2 = A^2(e_1^2 - 1) = (Ae_1)^2 - A^2$

$\Rightarrow B^2 = 1 - \sin^2\theta = \cos^2\theta$

$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \Rightarrow \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$ $\Rightarrow x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

69. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$ then assuming k as an integer,

(A) f(x) increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(B) f(x) decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(C) f(x) decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

(D) f(x) increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

Ans. **(AC)**

Sol. $f(x) = \cos\left(\frac{\pi}{x}\right)$

$f'(x) = -\sin\left(\frac{\pi}{x}\right)\left(-\frac{\pi}{x}\right) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$

for increasing function f'(x) > 0

$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$

$(2k\pi) < \frac{\pi}{x} < (2k+1)\pi$

$\frac{1}{2k} > x > \frac{1}{(2k+1)}$

for decreasing function f'(x) < 0

$\sin\left(\frac{\pi}{x}\right) < 0$

$\Rightarrow \frac{\pi}{x} \in ((2k+1)\pi, (2k+2)\pi) \Rightarrow x \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

70. Consider the function $y = \log_a\left(x + \sqrt{x^2+1}\right)$, $a > 0$, $a \neq 1$. The inverse of the function

(A) does not exist

(B) is $x = \log_{1/a}\left(y + \sqrt{y^2+1}\right)$

(C) is x = sinh (y ln a)

(D) is $x = \cosh\left(-y \ln\frac{1}{a}\right)$

Ans. **(C)**

Sol. $a^y = \left(x + \sqrt{x^2+1}\right)$

$\Rightarrow a^{-y} = \sqrt{x^2+1} - x$ $\quad\Rightarrow a^y - a^{-y} = 2x$

$\Rightarrow f^{-1}(y) = x = \frac{a^y - a^{-y}}{2} = \frac{e^{y\ell na} - e^{-y\ell na}}{2} = \sin h(y\,\ell na)$

$\left(\text{since } \sinh(x) = \frac{e^x - e^{-x}}{2}\right)$

71. Let $I = \int_0^1 \frac{x^3 \cos 3x}{2+x^2} dx$. Then

(A) $-\frac{1}{2} < I < \frac{1}{2}$ (B) $-\frac{1}{3} < I < \frac{1}{3}$ (C) $-1 < I < 1$ (D) $-\frac{3}{2} < I < \frac{3}{2}$

Ans. (ABCD)

Sol. $-1 < \cos 3x < 1$

$-x^3 < x^3 \cos 3x < x^3$

$$\frac{-x^3}{x^2} < -\frac{x^3}{x} < \frac{-x^3}{2+x^2} < \frac{x^3 \cos 3x}{2+x^2} < \frac{x^3}{2+x^2} < \frac{x^3}{x} < \frac{x^3}{x^2}$$

taking integration from 0 to 1

$$\Rightarrow \int_0^1 -x^2\, dx < I < \int_0^1 x^2\, dx$$

$$\Rightarrow \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1 \Rightarrow -\frac{1}{3} < I < \frac{1}{3}$$

So, (ABCD)

72. A particle is in motion along a curve $12y - x^3$. The rate of change of its ordinate exceeds that of abscissa in

(A) $-2 < x < 2$ (B) $x = \pm 2$ (C) $x < -2$ (D) $x > 2$

Ans. (CD)

Sol. Given $\frac{dy}{dt} > \frac{dx}{dt}$ ......(i)

and $12y = x^3$

$$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt} \quad ......(ii)$$

from (1) $3x^2 \frac{dx}{dt} > 12\frac{dx}{dt}$

$\Rightarrow x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

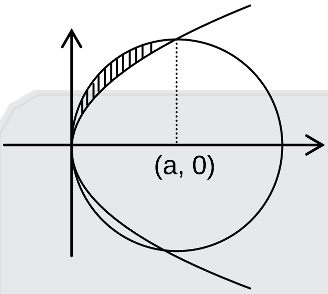
Ans. (CD)

73. The area of the region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ & the parabola $y^2 = ax$, $a > 0$ is

(A) $8\pi a^2$ (B) $a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$ (C) $\frac{16\pi a^2}{9}$ (D) $\pi\left(\frac{27}{8}+3a^2\right)$

Ans. **(B)**

Sol.



$(x-a)^2 + y^2 = a^2$

$\Rightarrow$ $x^2 + y^2 = 2ax$ and $y^2 = ax$

$\Rightarrow$ $x^2 + ax = 2ax$ $\Rightarrow$ $x^2 = ax$

$x = 0, x = a$

$(0, 0), (a, a)$

$$\text{Area} = \frac{1}{4}(\text{Area of circle}) - \int_0^a \sqrt{ax}\,dx = \frac{1}{4}(\pi a^2) - \sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^a = \frac{\pi a^2}{4} - \frac{2a^2}{3} = a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$$

Ans. (B)

74. If the equation $x - cx + d = 0$ has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

(A) both real (B) both negative

(C) both positive (D) one positive and one negative

Ans. **(AD)**

Sol. Let $x^2 + ax + b = 0$ has roots α and β

$x^2 - cx + d = 0$, roots are α^4 and β^4

$\alpha + \beta = -a$, $\alpha\beta = b$ and $\alpha^4 + \beta^4 = c$, $(\alpha\beta)^4 = d$

$\Rightarrow$ $b^4 = d$ and $\alpha^4 + \beta^4 = c$

$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow$ $((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow$ $(a^2 - 2b)^2 - 2b^2 = c$ $\Rightarrow$ $2b^2 + c = (a^2 - 2b)^2$

$2b^2 - c = 4a^2b - a^2$

$= a^2(4b - a^2)$

Now for equation

$x^2 - 4bx + 2b^2 - c = 0$

$D = (4b)^2 - 4(1)(2b^2 - c)$

$= 16b^2 - 8b^2 + 4c$

$= 8b^2 + 4c$

$= 4(2b^2 + c)$

$= 4(a^2 - 2b)^2 > 0 \Rightarrow$ real roots

Now $f(0) = 2b^2 - c$

$= a^2(4b - a^2)$

< 0 (since $a^2 > 4b$)

Roots are opposite in sign.

75. On the occasion of Dipawali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards send by students is

(A) ${}^{20}C_2$ (B) ${}^{20}P_2$ (C) $2 \times {}^{20}C_2$ (D) $2 \times {}^{20}P_2$

Ans. **(BC)**

Sol. Number of ways $= {}^{20}C_2 \times 2!$

$= {}^{20}P_2$