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SUBJECT : MATHEMATICS

WEST BENGAL JOINT ENTRANCE EXAMINATION (WBJEE) 2019

Date: 26 May, 2019 | Duration: 2 Hours | Max. Marks: 100

:: IMPORTANT INSTRUCTIONS ::

1. This question paper contains all objective questions divided into three categories. Each question has four answer options given.
2. **Category-I :** Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer ¼ marks will be deducted.
3. **Category-II :** Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer ½ marks will be deducted.
4. **Category-III :** Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong but there is no negative marking for the same and zero marks will be awarded.
5. Questions must be answered on, OMR sheet by darkening the appropriate bubble marked (A), (B), (C) or (D).
6. Use only Black/Blue ball point pen to mark the answer by complete filing up of the respective bubbles.
7. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
8. Write question booklet number and your roll number carefully in the specified locations of the OMR. Also fill appropriate bubbles.
9. Write your name (in block letter), name of the examination centre and put you full signature in appropriate boxes in the OMR.
10. The OMRs will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination center or signature of the candidate vis-à-vis what is given in the candidate's admit card. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
11. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will reported against & his/her candidature will be summarily cancelled.
12. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
13. Hand over the OMR to the invigilator before leaving the Examination Hall.
14. This paper contains questions in both English and Bengali. Necessary care and precaution were taken while framing the Bengali version. However if any discrepancy(ies) is/are found between the two versions, the information provided in the English version will stand and will be treated as final.

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MATHEMATICS

(Category-1 (Q.1 to Q.50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer ¼ mark will be deducted.

1. P is the extremity of the latusrectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is

(A) $\frac{\pi}{8}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans. **(C)**

Sol. Equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

its parametric coordinate are $\left(4\cos\theta, 2\sqrt{3}\sin\theta\right)$

P is (2, 3)

$\Rightarrow \cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

2. The direction ratios of the normal of the plane passing through the points (1,2,3),(–1,–2,1) and parallel to $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}$ is

(A) (2,3,4) (B) (14, –8, –1) (C) (–2, 0, –3) (D) (1, –2, –3)

Ans. **(B)**

Sol. Normal vector of plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1+1 & 2+2 & -3-1 \end{vmatrix}$

$= -28\hat{i} + 16\hat{j} + 2\hat{k}$

Direction ratios of the normal to plane can be (14, –8, –1)

3. The equation of the plane, which bisects the line joining the points (1,2,3) and (3,4,5) at right angles is

(A) $x + y + z = 0$ (B) $x + y - z = 9$ (C) $x + y + z = 9$ (D) $x + y - z + 9 = 0$

Ans. **(C)**

Sol. Equation of required plane is $(3 - 1)x + (4 - 2)y + (5 - 3)z = k$ which passes through (2, 3, 4)

$\Rightarrow k = 9$

$\Rightarrow$ equation of plane is $x + y + z = 9$

4. The limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is

(A) π (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{2\pi}{3}$

Ans. **(A)**

Sol. $\lim\limits_{n\to\infty} \frac{(n-2)\pi}{n} = \pi =$ required

5. Let f(x) > 0 for all x and f ′(x) exists for all x. If f is the inverse function of h and $h'(x)=\frac{1}{1+\log x}$. Then f ′(x) will be

(A) 1 + log (f(x)) (B) 1 + f(x) (C) 1 – log (f(x)) (D) log f(x)

Ans. **(A)**

Sol. $f'(x) \times h'(f(x)) = 1$

$\Rightarrow f'(x) = \frac{1}{h'(f(x))} = 1 + \log f(x)$

6. Consider the function f(x) = cosx^2. Then

(A) f is of period 2π (B) f is of period $\sqrt{2\pi}$ (C) f is not periodic (D) f is of periodic π

Ans. **(C)**

Sol. Let f(x) is periodic with period equal to T

then $\cos(x+T)^2 = \cos x^2 \ \forall x \in R$

$\Rightarrow -2\sin\left(\frac{(x+T)^2-x^2}{2}\right)\sin\left(\frac{(x+T)^2+x^2}{2}\right) = 0 \ \forall x \in R$

$\Rightarrow (x+T)^2 - x^2 = n\pi$ or $(x+T)^2 + x^2 = n\pi \ \forall x \in R$

which is false because these equation are quadratic equation but not identity

$\Rightarrow$ f(x) is not periodic

7. $\lim\limits_{x\to 0^+}(e^x + x)^{1/x}$

(A) Does not exist finitely (B) is 1

(C) is e^2 (D) is 2

Ans. **(C)**

Sol. $\lim\limits_{x\to 0^+}(e^x + x)^{1/x} = e^{\lim\limits_{x\to 0^+}\left(\frac{e^x-1+x}{x}\right)} = e^{1+1} = e^2$

8. Let f(x) be a derivable function, f ′(x) > f(x) and f(0) = 0. Then

(A) f(x) > 0 for all x > 0 (B) f(x) < 0 for all x > 0

(C) no sign of f(x) can be ascertained (D) f(x) is a constant function

Ans. **(A)**

Sol. $e^{-x}(f'(x) - f(x)) > 0$

$\Rightarrow (e^{-x}f(x))' > 0$ $\Rightarrow e^{-x}f(x)$ is increasing function

$\Rightarrow e^{-x}f(x) > e^{-0}f(0) \ \forall x > 0$ $\Rightarrow f(x) > 0 \ \forall x > 0$

9. Let f : [1, 3] → R be a continuous function that is differentiable in (1, 3) and $f'(x) = |f(x)|^2 + 4$ for all $x \in (1, 3)$. Then

(A) f(3) – f(1) = 5 is true (B) f(3) – f(1) = 5 is false

(C) f(3) – f(1) = 7 is false (D) f(3) – f(1) < 0 only at one point of (1, 3)

Ans. **(B,C)**

Sol. $\frac{f(3)-f(1)}{3-1} = f'(c)$ for atleast one $c \in (1, 3)$

$\Rightarrow f(3) - f(1) = 2(f(c))^2 + 8$ for atleast one $c \in (1, 3)$

$\Rightarrow f(3) - f(1) \geq 8$

$\Rightarrow f(3) - f(1) \neq 5$

Similarly $f(3) - f(1) \neq 7$

10. $\lim\limits_{x\to 0^+}(x^n \ell nx),\ n > 0$

(A) does not exist
(B) exists and is zero
(C) exists and is 1
(D) exists and is e^{-1}

Ans. **(B)**

Sol. $\lim\limits_{x\to 0^+}(x^n \ell nx)$

$= \lim\limits_{x\to 0^+}\left(\dfrac{\ell nx}{1/x^n}\right) = \lim\limits_{x\to 0^+}\left(\dfrac{1/x}{\frac{-n}{x^{n+1}}}\right)$ (using L hospital rule)

$= \lim\limits_{x\to 0^+}\left(\dfrac{x^n}{-n}\right) = 0$

11. If $\int \cos x \log\left(\tan\frac{x}{2}\right)dx = \sin x \log\left(\tan\frac{x}{2}\right) + f(x)$ then $f(x)$ is equal to, (assuming c is a arbitrary real constant)

(A) c
(B) c – x
(C) c + x
(D) 2x + c

Ans. **(B)**

Sol. $\int \cos x \log\left(\tan\frac{x}{2}\right)dx = \sin x \log\left(\tan\frac{x}{2}\right) - \int \sin x\left(\frac{1}{\sin x}\right)dx$

$= \sin x\left(\tan\frac{x}{2}\right) - x + c$

$\Rightarrow f(x) = c - x$

12. $y = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is an equation of a family of

(A) straight lines
(B) circles
(C) ellipses
(D) parabolas

Ans. **(D)**

Sol. $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

so $\int \cos\left(2\left(\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right)dx = \int \dfrac{1-\left(\frac{1-x}{1+x}\right)}{1+\frac{1-x}{1+x}}dx$

$= \int x\,dx = \dfrac{x^2}{2} + c$

$\Rightarrow y = \dfrac{x^2}{2} + c$ are family of parabolas

13. The value of the integration $\int_{-\pi/4}^{\pi/4}\left(\lambda|\sin x| + \dfrac{\mu \sin x}{1+\cos x} + \gamma\right)dx$

(A) is independent of λ only
(B) is independent of μ only
(C) is independent of γ only
(D) depends on λ, μ and γ

Ans. **(B)**

Sol. $\int_{-\pi/4}^{\pi/4}\left(\lambda|\sin x|+\frac{\mu\sin x}{1+\cos x}+\gamma\right)dx$

$= 2\lambda\int_{0}^{\pi/4}\sin x+0+\frac{\gamma\pi}{2}=2\lambda\left(1-\frac{1}{\sqrt{2}}\right)+\frac{\gamma\pi}{2}$

which is independent of μ

14. The value of

$\lim_{x\to 0}\frac{1}{x}\left[\int_{y}^{a}e^{\sin^2 t}dt-\int_{x+y}^{a}e^{\sin^2 t}dt\right]$ is equal to

(A) $e^{\sin^2 y}$ (B) $e^{2\sin y}$ (C) $e^{|\sin y|}$ (D) $e^{\operatorname{cosec}^2 y}$

Ans. **(A)**

Sol. $\lim_{x\to 0}\left[\int_{y}^{a}e^{\sin^2 t}dt-\int_{a}^{x+y}e^{\sin^2 t}dt\right]$

$= \lim_{x\to 0}\frac{\int_{y}^{x+y}e^{\sin^2 t}dt}{x} = \lim_{x\to 0}\frac{e^{\sin^2}(y+x)}{1}$ (using L hospital rule)

$= e^{\sin^2 y}$

15. If $\int 2^{2^x}.2^x\,dx = A.2^{2^x}+c$, then A =

(A) $\frac{1}{\log 2}$ (B) log2 (C) $(\log 2)^2$ (D) $\frac{1}{(\log 2)^2}$

Ans. **(D)**

Sol. $I = \int 2^{2^x}.2^x\,dx = \int 2^{2^x}.(2^x\ell n2)\times\frac{(\ell n2)}{(\ell n2)^2}dx$

put $2^{2^x}=t \Rightarrow (2^{2^x}\ell n2)(2^x\ell n2) = dt \Rightarrow I = \int\frac{dt}{(\ell n2)^2}=\frac{t}{(\ell n2)^2}+c=\frac{2^{2^x}}{(\ell n2)^2}+c \Rightarrow A = \frac{1}{(\log 2)^2}$

16. The value of the integral $\int_{-1}^{1}\left\{\frac{x^{2015}}{e^{|x|}(x^2+\cos x)}+\frac{1}{e^{|x|}}\right\}dx$ is equal to

(A) 0 (B) $1 - e^{-1}$ (C) $2e^{-1}$ (D) $2(1 - e^{-1})$

Ans. **(D)**

Sol. $\int_{1}^{1}\left\{\frac{x^{2015}}{e^{|x|}(x^2+\cos x)}+\frac{1}{e^{|x|}}\right\}dx = 0 + \int_{-1}^{1}\frac{dx}{e^{|x|}} = 2\int_{0}^{1}e^{-x}dx = 2\left|-e^{-x}\right|_0^1 = 2\left(-\frac{1}{e}+1\right) = 2(1-e^{-1})$

17. $\lim_{n\to\infty}\frac{1}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+....+\sqrt{\frac{n}{n+3(n-1)}}\right\}$

(A) does not exist (B) is 1 (C) is 2 (D) is 3

Ans. **(C)**

Sol. Required = $\lim_{n\to\infty}\frac{3}{n}\left\{\sqrt{\frac{1}{1+3\left(\frac{0}{n}\right)}}+\sqrt{\frac{1}{1+3\left(\frac{1}{n}\right)}}+....+\sqrt{\frac{1}{1+3\left(\frac{n-1}{n}\right)}}\right\}$

$= 3\int_0^1\sqrt{\frac{1}{1+3x}}dx = 3\left|\frac{(1+3x)^{1/2}}{\frac{1}{2}\times 3}\right|_0^1 = 3\times\frac{2}{3}\left(4^{1/2}-1^{1/2}\right)=\frac{2}{3}\times 3 = 2$

18. The general solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{\frac{x}{y}}dy=0$ is (c is an arbitrary constant)

(A) $x - ye^{\frac{x}{y}} = c$ (B) $y - xe^{\frac{x}{y}} = c$ (C) $x + ye^{\frac{x}{y}} = c$ (D) $y + xe^{\frac{x}{y}} = c$

Ans. **(C)**

Sol. $dx + e^{x/y}dx + e^{x/y}dy - \frac{x}{y}e^{x/y}dy = 0$

$\Rightarrow dx + e^{x/y}dy + e^{x/y}\frac{(ydx - xdy)}{y} = 0$

$\Rightarrow dx + e^{x/y}dy + yd(e^{x/y}) = 0$

$\Rightarrow dx + d(ye^{x/y}) = 0$

$\Rightarrow x + ye^{x/y} = c$

19. General solution of $(x + y)^2\frac{dy}{dx} = a^2$, $a \neq 0$ is (c is arbitrary constant)

(A) $\frac{x}{a} = \tan\frac{y}{a} + c$ (B) $\tan xy = c$ (C) $\tan(x + y) = c$ (D) $\tan\frac{y+c}{a} = \frac{x+y}{a}$

Ans. **(D)**

Sol. $\frac{dy}{dx} = \frac{a^2}{(x+y)^2}$

put $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} = 1 + \frac{a^2}{t^2} \Rightarrow \frac{t^2}{t^2+a^2}dt = dx \Rightarrow dt - \frac{a^2dt}{t^2+a^2} = dx$

$\Rightarrow t - a\tan^{-1}\frac{t}{a} + c = x \Rightarrow y + c = a\tan^{-1}\frac{x+y}{a} \Rightarrow \tan\left(\frac{y+c}{a}\right) = \frac{x+y}{a}$

20. Let P(4, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at P intersects the X-axis at (16, 0), then the eccentricity of the hyperbola is

(A) $\frac{\sqrt{5}}{2}$ (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans. **(B)**

Sol. $\frac{dy}{dx}=\frac{b^2x}{a^2y}$

$$-\left.\frac{dx}{dy}\right]_{(4,3)}=-\frac{3a^2}{4b^2}=\frac{3-0}{4-16}$$

$$\Rightarrow \frac{a^2}{b^2}=\frac{1}{3}\Rightarrow \frac{b^2}{a^2}=3$$

$$e=\sqrt{\frac{1+b^2}{a^2}}=\sqrt{1+3}=2$$

21. If the radius of a spherical balloon increases by 0.1% then its volume increases approximately by

(A) 0.2% (B) 0.3% (C) 0.4% (D) 0.05%

Ans. **(B)**

Sol. $$\frac{\Delta V}{V}\times 100=\frac{\left(\frac{4}{3}\pi\left(r+\frac{r}{1000}\right)^3-\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3}\times 100=\left(\left(1+\frac{1}{1000}\right)^3-1\right)100$$

$$=\frac{3}{10}+\frac{3}{10000}+\frac{1}{10000000}\simeq 0.3\%\text{ approx.}$$

22. The three sides of a right-angled triangle are in G.P. (geometrical progression). If the two acute angles be α and β, then $\tan\alpha$ and $\tan\beta$ are

(A) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$ (B) $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$

(C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

Ans. **(B)**

Sol. Let sides are a, ar, ar^2 (r > 1)

Then $a^2+a^2r^2=a^2r^4 \Rightarrow r^4+r^2-1=0\Rightarrow r^2=\frac{1+\sqrt{5}}{2} \Rightarrow r=\sqrt{\frac{1+\sqrt{5}}{2}}$

and $\frac{1}{r}=\sqrt{\frac{2}{\sqrt{5}+1}}=\sqrt{\frac{\sqrt{5}-1}{2}}$

$\Rightarrow \tan\alpha, \tan\beta$ equal to r or $\frac{1}{r}$

$\Rightarrow \tan\alpha, \tan\beta=\sqrt{\frac{\sqrt{5}+1}{2}}$ or $\sqrt{\frac{\sqrt{5}-1}{2}}$

23. If $\log_2 6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$, then the value of x are

(A) $\frac{1}{4}, \frac{1}{3}$ (B) $\frac{1}{4}, \frac{1}{2}$ (C) $-\frac{1}{4}, \frac{1}{2}$ (D) $\frac{1}{3}, -\frac{1}{2}$

Ans. **(B)**

Sol. $\log_2(2^{1/x} + 8) - \log_2 6 = \frac{1}{2x}$

$\Rightarrow \log_2\left(\frac{2^{1/x}+8}{6}\right) = \frac{1}{2x}$

$\Rightarrow \frac{2^{1/x}+8}{6} = 2^{1/2x}$

$\Rightarrow (2^{1/2x})^2 - 6(2^{1/2x}) + 8 = 0$

$\Rightarrow 2^{1/2x} = 2, 4$

$\Rightarrow 2x = 1, \frac{1}{2}$

$\Rightarrow x = \frac{1}{2}, \frac{1}{4}$

24. Let z be a complex number such that the principal value of argument, arg z > 0. Then arg z – arg(–z) is

(A) $\frac{\pi}{2}$ (B) $\pm\pi$ (C) π (D) $-\pi$

Ans. **(C)**

Sol. If arg $z = \theta > 0$

then arg $(-z) = \theta - \pi$

Now arg $z - \arg(-z) = \theta - (\theta - \pi) = \pi$

25. The general value of the real angle θ, which satisfies the equation,

$(\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)......(\cos n\theta + i\sin\theta) = 1$ is given by (assuming k is an integer)

(A) $\frac{2k\pi}{n+2}$ (B) $\frac{4k\pi}{n(n+1)}$ (C) $\frac{4k\pi}{n+1}$ (D) $\frac{6k\pi}{n(n+1)}$

Ans. **(B)**

Sol. $e^{i\theta}, e^{i2\theta}$ .... $e^{in\theta} = 1$

$\Rightarrow e^{i\theta\frac{n(n+1)}{2}} = 1 \Rightarrow \frac{n(n+1)\theta}{2} = 2k\pi, k \in I$

$\theta = \frac{4k\pi}{n(n+1)}, k \in I$

26. Let a,b,c be real numbers such a + b + c < 0 and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

(A) a > 0, c > 0 (B) a > 0, c < 0 (C) a < 0, c > 0 (D) a < 0, c < 0

Ans. **(D)**

Sol. D < 0 and f(1) < 0

$\Rightarrow f(x) < 0 \ \forall\ x \in R$ and $a < 0$

$\Rightarrow f(0) < 0 \Rightarrow c < 0$

27. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions?

(A) 850 (B) 800 (C) 750 (D) 700

Ans. **(A)**

Sol. ${}^6C_2 \times {}^6C_4 + {}^6C_3 \times {}^6C_3 + {}^6C_4 \times {}^6C_2 = (12)^2 + (20)^2 + (15)^2 = 225 + 400 + 225 = 850$

28. There are 7 greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is

(A) ${}^7C_3$ (B) $2.{}^7C_3$ (C) $3!\,{}^4C_4$ (D) $3!\,{}^7C_3\,{}^4C_3$

Ans. **(B)**

Sol. ${}^7C_4 \times$ (De-arrangement of 3 things) $\Rightarrow 35 \times 2 = 70 = {}^7C_3 \times 2$

29. $7^{2n} + 16n - 1$ $(n \in N)$ is divisible by

(A) 65 (B) 63 (C) 61 (D) 64

Ans. **(D)**

Sol. $7^{2n} + 16n - 1 = (8-1)^{2n} + 16n - 1 = 64\lambda - 16n + 1 + 16n - 1 = 64\lambda$

30. The number of irrational terms in the expansion of $\left(3^{\frac{1}{8}} + 5^{\frac{1}{4}}\right)^{84}$ is

(A) 73 (B) 74 (C) 75 (D) 76

Ans. **(B)**

Sol. $(3^{1/8} + 5^{1/4})^{84} = (5^{1/4} + 3^{1/8})^{84}$

Now $T_n = {}^{84}C_n \left(5^{\frac{84-n}{4}}\right)(3^{n/8})$

If T_n = rational $\Rightarrow$ n is multiple of 8

$\Rightarrow n = 0, 8, 16, \ldots\ldots 80 \Rightarrow$ n can take 11 terms $\Rightarrow$ number of rational terms = 11

$\Rightarrow$ number of irrational terms = 85 – 11 = 74

31. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix $A - 3I_3$ is

(A) invertible (B) orthogonal (C) non-invertible (D) real skew symmetric matrix

Ans. **(C)**

Sol. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A - 3I_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$\det(A - 3I_3) = -2(3) + 1(3) + 1(3) = 0 \quad \Rightarrow A - 3I_3$ is non invertible

32. If M is any square matrix of order 3 over R and If M′ be the transpose of M, then adj(M′) – adj(M)′ is equal to

(A) M (B) M′ (C) null matrix (D) identity matrix

Ans. **(C)**

Sol. for square matrix adj(M′) = (ads M)′ so adj(M′) – (adj M)′ = O = null matrix

33. If $A = \begin{pmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{pmatrix}$ and $|A^2| = 25$, then $|x|$ is equal to

(A) $\frac{1}{5}$ (B) 5 (C) 5^2 (D) 1

Ans. **(A)**

Sol. $|A^2| = 25 \Rightarrow |A| = 5$ or -5

now $|A| = 25$ x = 5 or $-5 \Rightarrow x = \frac{1}{5}, -\frac{1}{5}$ $\Rightarrow |x| = \frac{1}{5}$

34. Let A and B be two square matrices of order 3 and $AB = O_3$, where O_3 denotes the null matrix of order 3. Then

(A) must be $A = O_3$, $B = O_3$
(B) if $A \neq O_3$, must be $B \neq O_3$
(C) if $A = O_3$, must be $B \neq O_3$
(D) may be $A \neq O_3$, $B \neq O_3$

Ans. **(D)**

Sol. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Here AB = O but $A \neq O$, $B \neq O$
so if AB = O then may be $A \neq O$, B = O

35. Let P and T be the subsets of X-Y plane defined by

$$P = \{(x,y) : x > 0, y > 0 \text{ and } x^2 + y^2 = 1\}$$
$$T = \{(x,y) : x > 0, y > 0 \text{ and } x^8 + y^8 < 1\}$$

Then $P \cap T$ is

(A) the void set ϕ (B) P (C) T (D) $P-T^C$

Ans. **(B)**

Sol. Let (h, k) satisfies $x^2 + y^2 = 1$ then $h^2 + k^2 = 1$
Now $h^8 + k^8 = h^8 + (1 - h^2)^4 = 2h^8 - 4h^6 + 6h^4 - 4h^2 + 1 = 2h^2(h^2 - 1)(h^4 - h^2 + 2) + 1$
$= -2h^2k^2(h^4 - h^2 + 2) + 1 < 1 \ \forall\ h > 0, k > 0$
$\Rightarrow$ all solution of $x^2 + y^2 = 1$ satisfies $x^8 + y^8 < 1$ $\Rightarrow P \cap T = P$

36. Let $f : R \to R$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in R$. Then

(A) f is one-one but not onto mapping
(B) f is onto but not one-one mapping
(C) f is both one-one and onto
(D) f is neither one-one nor onto

Ans. **(D)**

Sol. $f(x) = \frac{x^4}{1+x^2}$

f(x) is even function hence it is many one
Also $f(x) \geq 0\ \forall\ x \in R$, hence it is into function $\Rightarrow$ f(x) is neither one-one nor onto

37. Let the relation ρ be defined on R as $a\rho b$ iif $1 + ab > 0$. Then

(A) ρ is reflexive only
(B) ρ is equivalence relation
(C) ρ is reflexive and transitive but not symmetric
(D) ρ is reflexive and symmetric but not transitive

Ans. **(D)**

Sol. $(a, a) \in \rho$ because $1 + a^2 > 0$ $\Rightarrow \rho$ is reflexive

If $1 + ab > 0$ then $1 + ba > 0$ $\Rightarrow$ If $(a, b) \in \rho$ then $(b, a) \in \rho$ $\Rightarrow \rho$ is symmetric

Now $\left(-2, \frac{1}{8}\right) \in \rho$ and $\left(\frac{1}{8}, 10\right) \in \rho$

but $(-2, 10) \notin \rho$, hence ρ is not transitive.

38. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. Then probability that the problem will be solved at least by one student is

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{3}{4}$

Ans. **(C)**

Sol. Probability that no student solve the problem is

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$$

$\Rightarrow$ Probability that the problem will be solved by at least one student is equal to $1 - \frac{1}{5} = \frac{4}{5}$

39. If X is a random variable such that $\sigma(X) = 2.6$, then $\sigma(1 - 4X)$ is equal to

(A) 7.8 (B) –10.4 (C) 13 (D) 10.4

Ans. **(D)**

Sol. $\sigma(ax + b) = |a|\ (\sigma(x))$

so $\sigma(1 - 4x) = |-4|\ \sigma(x) = 4 \times 2.6 = 10.4$

40. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is

(A) 0 (B) 1 (C) 2 (D) 3

Ans. **(A)**

Sol. $e^{\sin x} - e^{-\sin x} = 4$

L.H.S in always less than 4

hence no solution exist.

41. The angles of a triangle are in the ratio 2 : 3 : 7 and the radius of the circumscribed circle is 10 cm. The length of the smallest side is

(A) 2 cm (B) 5 cm (C) 7cm (D) 10 cm

Ans. **(D)**

Sol. Let angles are $2x, 3x, 7x$ $\Rightarrow$ $12x = 180$ $\Rightarrow$ $x = 15$

angles are 30°, 45°, 75°

Let smallest side is a $\Rightarrow \frac{a}{\sin 30°} = 2(10)$ $\Rightarrow$ $a = 10$

42. A variable line passes through a fixed point (x_1, y_1) and meets the axes at A and B. If the rectangle OAPB be completed, the locus of P is, (O being the origin of the system of axes)

(A) $(y - y_1)^2 = 4(x - x_1)$ (B) $\frac{x_1}{x} + \frac{y_1}{y} = 1$

(C) $x^2 + y^2 = x_1^2 + y_1^2$ (D) $\frac{x^2}{2x_1^2} + \frac{y^2}{y_1^2} = 1$

Ans. **(B)**

Sol. Let P is (h, k) then A is (h, 0) and B is (0, k)

equation of AB is $\frac{x}{h}+\frac{y}{k}=1$ which passes through (x_1, y_1) $\Rightarrow$ $\frac{x_1}{h}+\frac{y_1}{k}=1$

$\Rightarrow$ $\frac{x_1}{x}+\frac{y_1}{y}=1$

43. A straight line through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}$ x + y = 1. If it intersects the X-axis, then its equation will be

(A) $y + x\sqrt{3} + 2 + 3\sqrt{3} = 0$ (B) $y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$

(C) $y - x\sqrt{3} - 2 - 2\sqrt{3} = 0$ (D) $x - x\sqrt{3} + 2 - 3\sqrt{3} = 0$

Ans. **(B)**

Sol. Angle made by line $\sqrt{3}$ x + y = 1 with positive x-axis in anti-clockwise direction is 120°. Now the required line makes either 180° or 60° angle with + x-axis. But required line is not parallel to x-axis. So slope of required line is tan60° = $\sqrt{3}$ $\Rightarrow$ required line is $(y + 2) = \sqrt{3}(x - 3)$ $\Rightarrow y - \sqrt{3}x + 2 + 3 = 0$

44. A variable line passes through the fixed point (α, β),. The locus of the foot of the perpendicular from the origin on the line is

(A) $x^2 + y^2 - \alpha x - \beta y = 0$ (B) $x^2 - y^2 + 2\alpha x + 2\beta y = 0$

(C) $\alpha x + \beta y \pm \sqrt{(\alpha^2+\beta^2)} = 0$ (D) $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

Ans. **(A)**

Sol. Let foot of origin on variable line is P

$\Rightarrow$ (0, 0) and (α, β) subtends right angle at P.

$\Rightarrow$ Locus of P is circle assuming (0, 0) & (α, β) as diameter.

$\Rightarrow$ Required locus is $(x - 0)(x - \alpha) + (y - 0)(y - \beta) = 0$

$\Rightarrow$ Required locus is $x^2 + y^2 - \alpha x - \beta y = 0$

45. if the point of intersection of the lines 2ax + 4ay + c = 0 and 7bx + 3by – d = 0 lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then ad : bc equals to

(A) 2 : 3 (B) 2 : 1 (C) 1 : 1 (D) 3 : 2

Ans. **(B)**

Sol. Let point of intersection of 2ax + 4ay + c = 0 and 7bx + 3by – d = 0 is (α, $-\alpha$)

$2a\alpha - 4a\alpha + c = 0$ and $7b\alpha - 3bd - d = 0$

$\alpha = c/2a = d/4b$ $\Rightarrow$ ad : bc = 2 : 1

46. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is

(A) $(x-p)^2 = 4qy$ (B) $(x-q)^2 = 4py$ (C) $(y-p)^2 = 4qx$ (D) $(y-q)^2 = 4px$

Ans. **(A)**

Sol. Let other end is (h, k), then centre equal to $\left(\frac{p+h}{2}, \frac{q+k}{2}\right)$

Because circle touches x-axis hence radius = $\left|\frac{q+k}{2}\right| \Rightarrow \sqrt{(h-p)^2+(k-q)^2} = 2\left|\frac{q+k}{2}\right|$

$\Rightarrow \quad (x-p)^2 = (y+q)^2 - (y-q)^2 \quad \Rightarrow \quad (x-p)^2 = 4qy$

47. If P(0, 0), Q(1, 0) and R $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

(A) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ (D) $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

Ans. **(C)**

Sol. Centre of circle can be incentre or excentres. Because ΔPQR is equilateral so incentre is same as centroid $\Rightarrow$ incentre is $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ = centre of circle

48. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains fixed when α varies ?

(A) Directrix (B) Vertices (C) foci (D) Eccentricity

Ans. **(C)**

Sol. Focus are $\left(\pm\sqrt{a^2+b^2}, 0\right) = \left(\pm\sqrt{\cos^2\alpha+\sin^2\alpha}, 0\right)$

$\Rightarrow$ Focus are $(\pm 1, 0)$ which is Independent of α $\Rightarrow$ Focus are fixed

49. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans. **(C)**

Sol. $b = \frac{\sqrt{3}}{2}(2ae)$

$b^2 = 3(a^2 - b^2)$

$4b^2 = 3a^2$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$e^2 = 1 \frac{b^2}{a^2} = 1 - \frac{3}{4} = \frac{1}{4} \quad \Rightarrow \quad e = 1/2$$

50. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

(A) $x = 3 \pm \sqrt{\frac{13}{6}}$ (B) $x = 3 \pm \sqrt{\frac{6}{13}}$ (C) $x = 6 \pm \sqrt{\frac{13}{3}}$ (D) $x = 6 \pm \sqrt{\frac{3}{13}}$

Ans. **(A)**

Sol. equation of hyperbola is $3(x^2 - 6x) - 3(y^2 - 4y) + 2 = 0$

$$\Rightarrow \quad 3(x-3)^2 - 3(y-2)^2 = -2 + 27 - 12 \quad \Rightarrow \quad (x-3)^2 - (y-3) = \frac{13}{3}$$

$$\Rightarrow \quad \frac{(x-3)^2}{(\sqrt{13/3})^2} - \frac{(y-3)^2}{(\sqrt{13/3})^2} = 1$$

$$\Rightarrow \quad \text{Equation of directrix are } x - 3 = \pm\frac{\sqrt{13/3}}{\sqrt{2}} \quad \Rightarrow \quad x = 3 \pm \frac{\sqrt{13}}{6}$$

Category-II (Q. 51 to Q. 65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

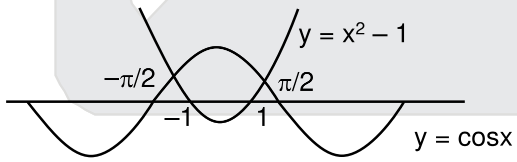
51. The graphs of the polynomial $x^2 - 1$ and $\cos x$ intersect

(A) at exactly two points (B) at exactly 3 points

(C) at least 4 but at finitely many points (D) at infinitely many points

Ans. **(A)**

Sol.



$y = x^2 - 1$ and $y = \cos x$ intersect at exactly two points

52. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly at a rate of 1 unit per second. Then, the rate of change of its ordinate, when the point passes through (5, 2)

(A) increases at the rate of $\frac{1}{2}$ unit per second

(B) decreases at the rate of $\frac{1}{2}$ unit per second

(C) decreases at the rate of $\frac{2}{5}$ unit per second

(D) increases at the rate of $\frac{2}{5}$ unit per second

Ans. **(C)**

Sol. $\frac{dy}{dt} = \frac{-10}{x^2}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{-10}{x^2} \Rightarrow \frac{dy}{dt}$ at x = 5 equal to $\frac{-10}{25}$

$\Rightarrow$ ordinate deceases at rate $\frac{2}{5}$ unit per second

53. Let a = min$\{x^2 + 2x + 3 : x \in R\}$ and $b = \lim\limits_{\theta \to 0}\frac{1-\cos\theta}{\theta^2}$. Then $\sum\limits_{r=0}^{n} a^r b^{n-r}$ is

(A) $\frac{2^{n+1}-1}{3.2^n}$ (B) $\frac{2^{n+1}+1}{3.2^n}$ (C) $\frac{4^{n+1}-1}{3.2^n}$ (D) $\frac{1}{2}(2^n-1)$

Ans. **(C)**

Sol. $(x^2 + 2x + 3) = (x + 1)^2 + 2 \Rightarrow$ min $(x^2 + 2x + 3)$ is 2 $\Rightarrow a = 2$

Now $\lim\limits_{\theta \to 0}\frac{1-\cos\theta}{\theta^2} = \frac{1}{2} \Rightarrow b = \frac{1}{2}$

$$\sum_{r=0}^{N} a^r b^{n-r} = b^n \sum_{r=0}^{n}\left(\frac{a}{b}\right)^r = \left(\frac{1}{2}\right)^n (1+4+4^2+......+4^n) = \frac{1}{2^n}\left(\frac{4^{n+1}-1}{4-1}\right) = \frac{4^{n+1}-1}{3.2^n}$$

54. Let a > b > 0 and $I(n) = a^{1/n} - b^{1/n}$, $J(n) = ((a - b)^{1/n}$ for all $n \ge 2$. then

(A) I(n) < J(n) (B) I(n) > J(n) (C) I(n) = J(n) (D) I(n) + J(n) = 0

Ans. **(A)**

Sol. If x > 0 and y > 0 then

$(x + y)^n > x^n + y^n$ when n > 1 and $(x + y)^n < x^n + y^n$ when 0 < n < 1

So $(x + y)^{1/n} < x^{1/n} + y^{1/n}$ when n > 1

Now assume x = b and y = a – b

Then $(b + (a - b))^{1/n} < b^{1/n} + (a - b)^{1/n} \Rightarrow a^{1/n} - b^{1/n} < (a - b)^{1/n}\ \forall n \ge 2$

$I(n) < J(n)\ \forall n \ge 2$

55. Let $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ be three unit vectors such that $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} \times \hat{\gamma})$ where $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = (\hat{\alpha}.\hat{\gamma})\hat{\beta} - (\hat{\alpha}.\hat{\beta})\hat{\gamma}$

If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans. **(D)**

Sol. $(\hat{\alpha}.\hat{\gamma})\hat{\beta} - (\hat{\alpha}.\hat{\beta})\hat{\gamma} = \frac{1}{2}\hat{\beta} + \frac{1}{2}\hat{\gamma}$ $\Rightarrow$ because $\hat{\beta}$ is not parallel to $\hat{\gamma}$ so $\hat{\alpha}.\hat{\beta} = -\frac{1}{2}$

$\Rightarrow$ angle between $\hat{\alpha}$ and $\hat{\beta}$ is $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

56. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + \lambda\hat{k}$ respectively. If the points A, B, C and D lie on a plane, the value of λ is

(A) 0 (B) 1 (C) 2 (D) – 4

Ans. **(D)**

Sol. vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ = Coplanar $\Rightarrow$ $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -1-\lambda \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ 0 & 0 & -4-\lambda \end{vmatrix} = 0 \Rightarrow (-4-\lambda)(-1+2) = 0 \Rightarrow \lambda = -4$$

57. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1, then it moves $\frac{1}{2}$ unit vertically up and reaches P_2, then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3, then it moves $\frac{1}{8}$ unit vertically down and reaches P_4, then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim\limits_{n\to\infty} x_n = \alpha$ and $\lim\limits_{n\to\infty} y_n = \beta$. Then (α, β) is

(A) (2, 3) (B) $\left(\frac{4}{3}, \frac{2}{5}\right)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) $\left(\frac{4}{3}, 3\right)$

Ans. **(B)**

Sol. $$\lim_{n\to\infty} x_n = \left(1 + 0 + \frac{1}{4} + 0 + \frac{1}{16} + ...\right) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\lim_{n\to\infty} y_n = \left(0 + \frac{1}{2} + 0 - \frac{1}{8} + 0 + \frac{1}{32} + ...\right) = \frac{1/2}{1+\frac{1}{4}} = \frac{2}{5} \quad \Rightarrow (\alpha, \beta) = \left(\frac{4}{3}, \frac{2}{5}\right)$$

58. For any non-zero complex number z, the minimum value of $|z| + |z - 1|$ is

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{3}{2}$

Ans. **(A)**

Sol. $|z| + |z - 1| \geq |z - (z - 1)|$ $\Rightarrow |z| + |z - 1| \geq 1$ $\Rightarrow$ minimum value of $|z| + |z - 1|$ is 1

59. The system of equations

$\lambda x + y + 3z = 0$

$2x + \mu y - z = 0$

$5x + 7y + z = 0$

has infinitely many solutions in R. Then,

(A) $\lambda = 2, \mu = 3$ (B) $\lambda = 1, \mu = 2$ (C) $\lambda = 1, \mu = 3$ (D) $\lambda = 3, \mu = 1$

Ans. **(C)**

Sol. $\begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0 \Rightarrow \lambda\mu + 7\lambda - 7 + 42 - 15\mu = 0 \Rightarrow (\lambda - 15)(\mu + 7) + 140 = 0$

$\Rightarrow (\lambda, \mu)$ can be equal to (1, 3)

60. Let $f : X \longrightarrow Y$ and A, B are non-void subsets of Y, then (where the symbols have their usual interpretation)

(A) $f^{-1}(A) - f^{-1}(B) \supset f^{-1}(A - B)$ but the opposite does not hold

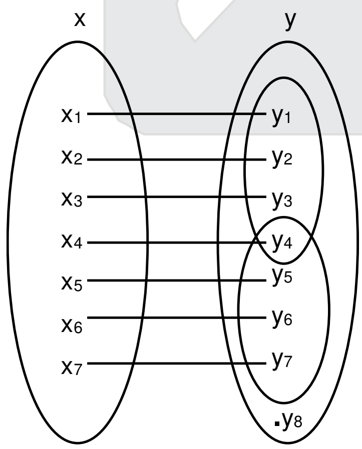
(B) $f^{-1}(A) - f^{-1}(B) \subset f^{-1}(A - B)$ but the opposite does not hold

(C) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$

(D) $f^{-1}(A - B) = f^{-1}(A) \cup f^{-1}(B)$

Ans. **(C)**

Sol. Direct formula. See example



$f^{-1}(A) - f^{-1}(B) = \{x_1, x_2, x_3\}$

$f^{-1}(A - B) = \{x_1, x_2, x_3\} \quad \Rightarrow \qquad f^{-1}(A) - f^{-1}(B) = f^{-1}(A - B)$

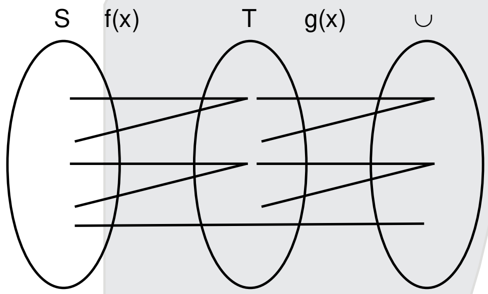
61. Let S, T, U be three non-void sets and $f : S \to T$, $g : T \to U$ be so that $g \circ f : S \to U$ is surjective. Then

(A) g and f are both surjective (B) g is surjective, f may not be so

(C) f is surjective, g may not be so (D) f and g both may not be surjective

Ans. **(B)**

Sol. Obvious g is surjective otherwise gof cannot be surjective but there is no need of f to be surjective. See example.



Hence f(x) is not surjective still gof is surjective

62. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$, The polar coordinate of the point Q, which is such that the line joining PQ is bisected perpendicularly by the initial line, is

(A) $\left(2, \frac{\pi}{4}\right)$ (B) $\left(2, \frac{\pi}{6}\right)$ (C) $\left(-2, \frac{\pi}{4}\right)$ (D) $\left(-2, \frac{\pi}{6}\right)$

Ans. **(A)**

Sol. If initial line is x-axis then Q is $\left(2, \frac{\pi}{4}\right)$

63. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then the eccentricity e is ,

(A) $= \sqrt{2}$ (B) $> \sqrt{2}$ (C) $< \sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans. **(B)**

Sol. $b > a \quad \Rightarrow \quad \frac{b^2}{a^2} > 1 \quad \Rightarrow \quad 1 + \frac{b^2}{a^2} > 2 \quad \Rightarrow \quad e^2 > 2 \quad \Rightarrow \quad e > \sqrt{2}$

64. The value of $\lim\limits_{x\to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$ is

(A) $\frac{[q]}{p}$ (B) 0 (C) 1 (D) ∞

Ans. (A)

Sol. $\lim\limits_{x\to 0^+} \frac{x}{p}\left[\frac{q}{x}\right] = \lim\limits_{x\to 0^+} \frac{x}{p}\left(\frac{q}{x} - \left\{\frac{q}{x}\right\}\right) = \lim\limits_{x\to 0^+}\left(\frac{q}{p} - \frac{x}{p}\left\{\frac{q}{x}\right\}\right) = \frac{q}{p} - (0 \times \text{finite}) = \frac{q}{p}$ (no answer is matched)

But B, C, D are totally wrong. Hence (A) may be correct.

65. Let $f(x) = x^4 - 4x^3 + 4x^2 + c$, $c \in \mathbb{R}$. Then

(A) f(x) has infinitely many zeros in (1, 2) for all c

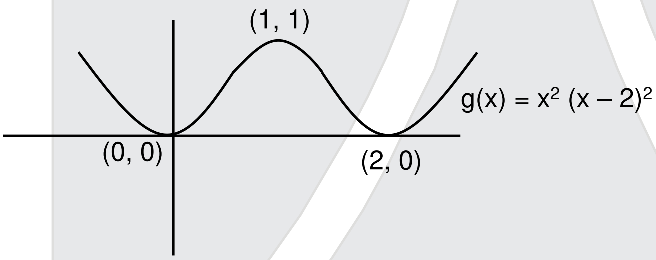
(B) f(x) has exactly one zero in (1, 2) if $-1 < c < 0$

(C) f(x) has double zeros in (1, 2) if $-1 < c < 0$

(D) whatever be the value of c, f(x) has no zero in (1, 2)

Ans. (B)

Sol.



$f(1) = 1 + c$

$f(2) = c$

$f(1)\,f(2) = c(1+c)$

$f(1)f(2) < 0 \Rightarrow c \in (-1, 0) \Rightarrow$ f(x) has exactly one zero in (1, 2) if $c \in (-1, 0)$

$f(x) = x^2(x-2)^2 + c$

Category-III (Q. 66 to Q. 75)

Carry 2 marks each and on or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked + actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let f and g be differentiable on the interval I and let a, b $\in$ I, a < b. Then

(A) If f(a) = 0 = f(b), the equation f'(x) + f(x)g'(x) = 0 is solvable in (a, b)
(B) If f(a) = 0 = f(b), the equation f'(x) + f(x)g'(x) = 0 may not be solvable in (a, b)
(C) If g(a) = 0 = g(b), the equation g'(x) + kg(x) = 0 is solvable in (a, b), k $\in$ R
(D) If g(a) = 0 = g(b), the equation g'(x) + kg(x) = 0 may not be solvable in (a, b), k $\in$ R

Ans. **(A,C)**

Sol. For option (A) & (B)
Let $h_1(x) = e^{g(x)} f(x)$
Now $h_1(a) = h_1(b) = 0$ and $h_1(x)$ is continuous also, so by Rolles theorem $h_1'(x) = 0$ has atleast one root in (a, b) $\Rightarrow e^{g(x)}(f'(x) + f(x)\,g'(x)) = 0$ has atleast one root in (a, b) $\Rightarrow$ Option (A) is correct.
Similarly assume $h_2(x) = e^{kx}\,g(x)$ for option (C) and (D) and apply same concept.

67. Consider the function $f(x) = \frac{x^3}{4} - \sin \pi x + 3$

(A) f(x) does not attain value within the interval [–2, 2]
(B) f(x) takes on the value $2\frac{1}{3}$ in the interval [–2, 2]
(C) f(x) takes on the value $3\frac{1}{4}$ in the interval [–2, 2]
(D) f(x) takes no value p, 1 < p < 5 in the interval [–2, 2]

Ans. **(B,C)**

Sol. f(–2) = 1 and f(2) = 5 and f is continuous also.
So intermediate value theorem, function f(x) takes all values between 1 to 5.
$\Rightarrow$ $2\frac{1}{3}$ and $3\frac{1}{4}$ lies in 1 to 5 so option B, C are correct

68. Let $I_n = \int_0^1 x^n \tan^{-1} x\, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

(A) a_1, a_2, a_3 are in G.P.
(B) b_1, b_2, b_3 are in A.P.
(C) c_1, c_2, c_3 are in H.P.
(D) a_1, a_2, a_3 are in A.P

Ans. **(B,D)**

Sol. $I_n = \left|\frac{x^{n+1}}{n+1}\tan^{-1} x\right|_0^1 - \int_0^1 \frac{x^{n+1}}{n+1}\left(\frac{1}{1+x^2}\right)dx$

$\Rightarrow (n+1)\, I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2}dx \quad \Rightarrow \quad (n+3)\, I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{1+x^2}dx$

$\Rightarrow (n+1)\, I_n + (n+3)\, I_{n+2} = \frac{\pi}{2} - \frac{1}{n+2} \quad \Rightarrow \quad a_n = n+1,\ b_n = n+3,\ c_n = \frac{\pi}{2} - \frac{1}{n+2}$

69. Two particles A and B move from rest along a straight line with constant accelerations f and h respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

(A) $(f + h)m^2 = fhn$ (B) $(f - fh)m^2 = fhn$ (C) $(h - f)n = \frac{1}{2}fhm^2$ (D) $\frac{1}{2}(f + h)n = fhm^2$

Ans. (C)

Sol. $S + n = \frac{1}{2}f(t + m)^2$ and $S = \frac{1}{2}ht^2$, $V = ht$

$\therefore \quad \frac{1}{2}ht^2 + n = \frac{1}{2}f(t + m)^2 \quad ....(I)$

Also $V = 0 + ht = 0 + f(t + m) \quad \Rightarrow \quad t + m = \frac{ht}{f}$

From equation (I), $\frac{1}{2}ht^2 + n = \frac{1}{2}f\left(\frac{ht}{f}\right)^2 \quad \Rightarrow \quad t^2 = \frac{2hf}{h(h-f)}$

Also,

$ht = f(t + m) \quad \Rightarrow \quad t^2 = \frac{m^2f^2}{(h-f)^2}$

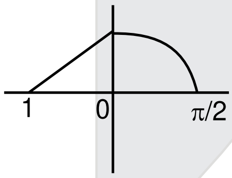
$\therefore \quad \frac{2nf}{h(h-f)} = \frac{m^2f^2}{(h-f)^2} \quad \Rightarrow \quad n(h - f) = \frac{1}{2}fhm^2$

70. The area bounded by $y = x + 1$ and $y = \cos x$ and the x-axis, is

(A) 1 sq. unit (B) $\frac{3}{2}$ sq. unit (C) $\frac{1}{4}$ sq. unit (D) $\frac{1}{8}$ sq. unit

Ans. **(B)**

Sol.



Area $\frac{1}{2} \times 1 \times 1 + \int_0^{\pi/2} \cos x\, dx = \frac{1}{2} + 1 = \frac{3}{2}$

71. let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P. then ab equals

(A) $\frac{24}{5}$ (B) 64 (C) 16 (D) 8

Ans. **(B)**

Sol. $x_1 + x_2 = 3$; $x_1 . x_2 = a$

$x_3 + x_4 = 12$; $x_3 . x_4 = b$

Let r be the common ratio of GP, then $\frac{x_3 + x_4}{x_1 + x_2} = \frac{x_1(1+r)}{x_1r^2(1+r)} = \frac{3}{12} \quad \Rightarrow \quad r = 2$ (G.P. increasing)

$\because \quad x_1 + x_2 = 3 \quad \Rightarrow \quad x_1(1 + r) = 3 \quad \Rightarrow \quad x_1 = 1$

$\therefore \quad ab = x_1x_2x_3x_4 = 1.2.4.8 = 64$

72. If $\theta \in R$ and $\frac{1-i\cos\theta}{1+2i\cos\theta}$ is real number, then θ will be (when I : Set of integers)

(A) $(2n+1)\frac{\pi}{2}, n \in I$ (B) $\frac{3n\pi}{2}, n \in I$ (C) $n\pi, n \in I$ (D) $2n\pi, n \in I$

Ans. (A)

Sol. $\because$ $\frac{1-i\cos\theta}{1+2i\cos\theta}$ is real $\Rightarrow$ $\frac{1-i\cos\theta}{1+2i\cos\theta} = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$\Rightarrow$ $1 - 3i\cos\theta - 2\cos^2\theta = 1 + 3i\cos\theta - 2\cos^2\theta$

$\Rightarrow$ $\cos\theta = 0$

$\Rightarrow$ $\theta = (2n+1)\frac{\pi}{2}$, $(n \in I)$

73. Let A = $\begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then the roots of the equation $\det(A - \lambda I_3) = 0$

(where I_3 is the identity matrix of order 3) are

(A) 3, 0, 3 (B) 0, 3, 6 (C) 1, 0, – 6 (D) 3, 3, 6

Ans. (B)

Sol. Let $(A - \lambda I_3) = 0 \Rightarrow$ $\begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow$ $(3-\lambda)^3 - 9(3-\lambda) = 0$ $\Rightarrow$ $(3-\lambda)[(3-\lambda)^2 - 3^2] = 0$

$\Rightarrow$ $3 - \lambda = 0$ or $3 - \lambda - 3 = 0$ or $3 - \lambda + 3 = 0$ $\Rightarrow$ $\lambda = 0, 3$ or 6

74. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that AB = AC. The equation of line AC passing through (2, –7) is

(A) $x - y - 9 = 0$ (B) $23x + 7y + 3 = 0$ (C) $2x - y - 11 = 0$ (D) $7x - 6y - 56 = 0$

Ans. (B)

Sol. If AB = AC $\Rightarrow \angle ABC = \angle ACB \Rightarrow \tan(\angle ABC) = \tan(\angle ACB)$

If let slope of AC is m

$\therefore$ $\left|\frac{m+\frac{1}{4}}{1-\frac{m}{4}}\right| = \left|\frac{-\frac{1}{4}-1}{1-\frac{1}{4}}\right|$ $\Rightarrow$ $m = \frac{-23}{7}$, 1 (rejected)

$\therefore$ Equation of line is $23x + 7y + 3 = 0$

75. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point (2, 8) is

(A) $3x - y + 2 = 0$ (B) $3x + y - 14 = 0$ (C) $23x - 3y - 22 = 0$ (D) $3x - 23y + 178 = 0$

Ans. (A,C)

Sol. Hyperbola is $\frac{x^2}{1} - \frac{y^2}{5} = 1$

Let the tangent be $y = mx \pm \sqrt{m^2 - 5}$

Since it passes through (2, 8) $\Rightarrow$ $(8 - 2m)^2 = m^2 - 5$ $\Rightarrow$ $m = 3$ or $\frac{23}{3}$